# Knowledge in Learning
# AIMA – Chapter 19

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http://boole.cs.iastate.edu/book/1-Science/1-ComputerScience/2-Book/Machine%20learning/

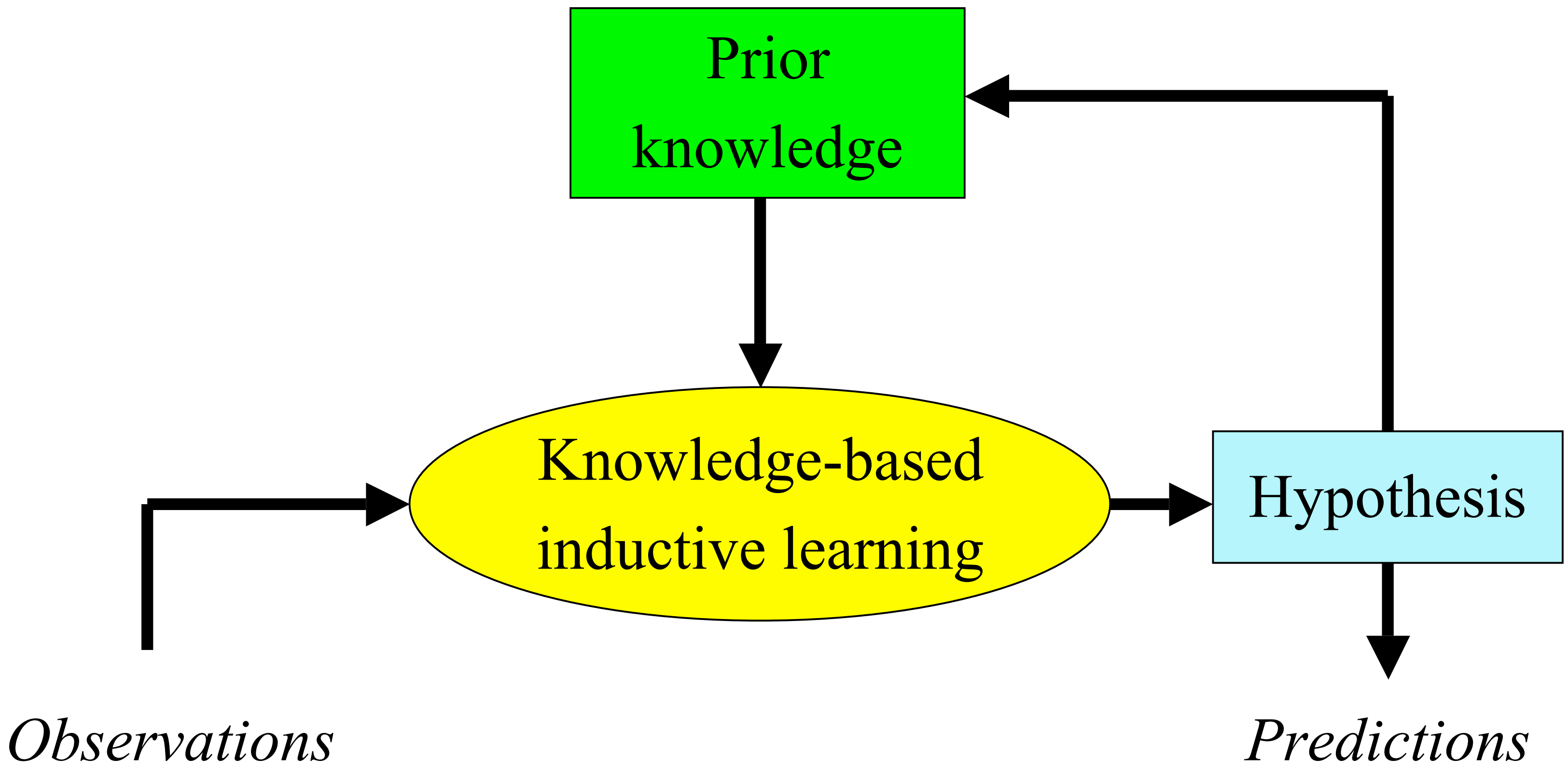
# Logical description of learning

- Examples are composed of descriptions and classifications
  - Object of inductive learning is to find a hypothesis that explains the classification of the examples, given their descriptions
- Entailment constraint

$$\mathit{Hypothesis} \wedge \mathit{Descriptions} \models \mathit{Classifications}$$

  - With *Descriptions* the conjunction of all the example descriptions and *Classifications* the conjunction of all the example classifications
  - *Example*: a decision tree that is consistent with all the examples will satisfy the entailment constraint
  - *Note*: Use Ockham's razor to avoid *Hypothesis* = *Classifications*

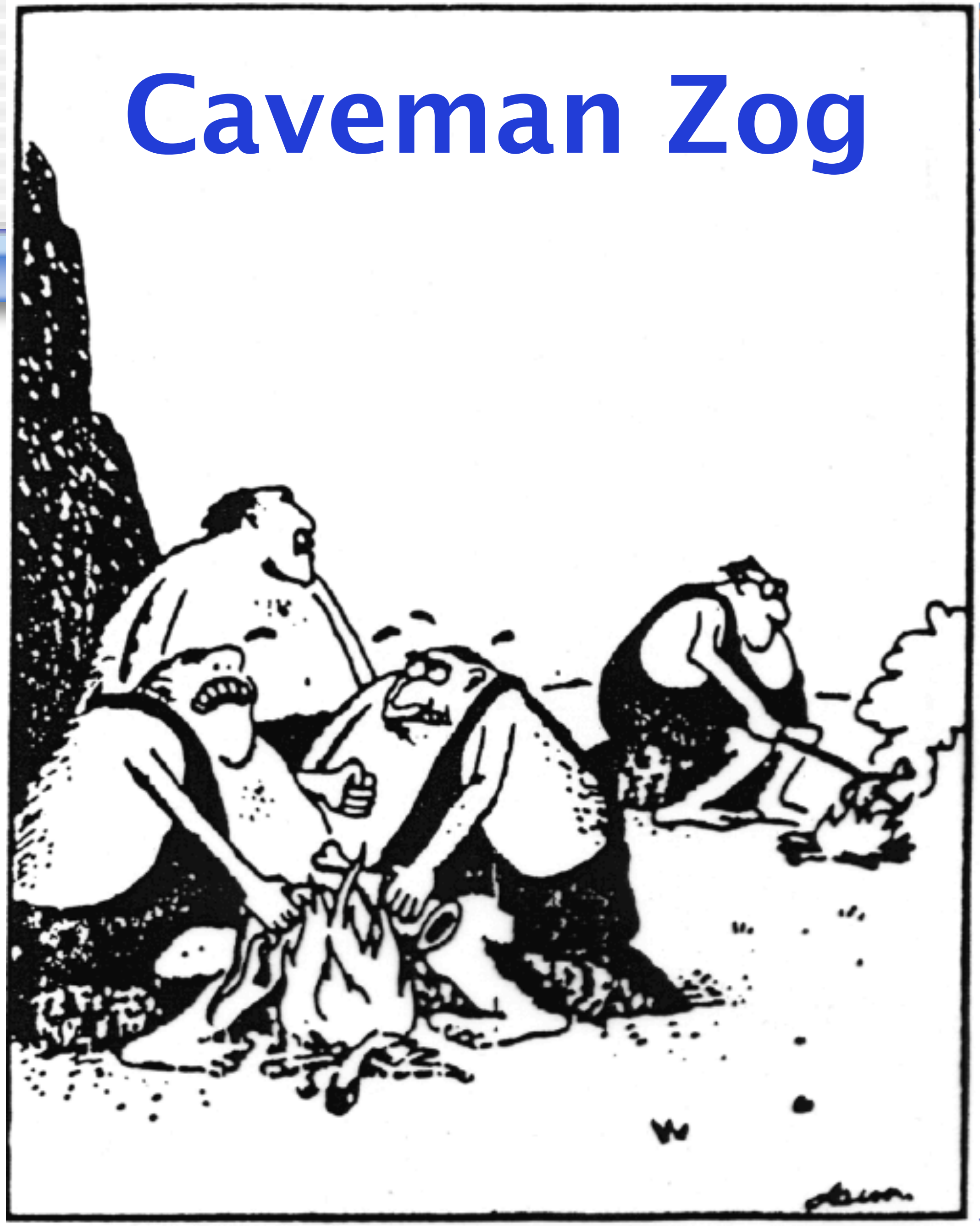
# Using prior knowledge



# Cumulative or Incremental Development

- To use background knowledge, a method to obtain background knowledge is needed
- This must be a learning process
- Use knowledge to learn more effectively
- *Question*: How to do this?
- *Examples where use of background knowledge is vital*
  - Caveman Zog and the lizard on a stick
  - Generalizing from one Brazilian
  - Density and conductance of copper can be generalized, but not mass
  - Inferring a general rule about antibiotic being effective for a particular type of infections

# Caveman Zog



"Hey!  Look what Zog do!"

# Adding Background Knowledge

- Explanation–based learning (EBL)
- Relevance–based learning (RBL)
- Knowledge–based inductive learning (KBIL)

# Explanation-based Learning

- Use explanation of success to infer a general rule
- General rule *follows logically* from the background knowledge

$$\textit{Hypothesis} \wedge \textit{Descriptions} \models \textit{Classifications}$$
$$\textit{Background} \models \textit{Hypothesis}$$

- *Does not learn anything factually new*
  - Converting first-principles theories into useful, special purpose knowledge

# Relevance-based Learning

- The prior knowledge concerns the *relevance* of a set of features to the goal predicate
- *Example*: In a given country most people speak the same language, but do not have the same name

*Hypothesis* ∧ *Descriptions* ⊨ *Classifications*
*Background* ∧ *Descriptions* ∧ *Classifications* ⊨ *Hypothesis*

- *Deductive learning*: Makes use of the observations, but does not produce hypothesis beyond the background knowledge and the observations

# Knowledge-based Inductive Learning

- The background knowledge and the new hypothesis combine to explain the examples
- *Example*
  - Inferring disease *D* from the symptoms is not enough to explain the prescription of medicine *M*
  - A rule that *M* is effective against *D* is needed

$$Background \land Hypothesis \land Descriptions \models Classifications$$

# Inductive Logic Programming

- Main field of study for KBIL algorithms
- Prior knowledge plays two key roles
  - The effective hypothesis space is reduced to include only those theories that are consistent with what is already known
  - Prior knowledge can be used to reduce the size of the hypothesis explaining the observations
    - Smaller hypotheses are easier to find
- *ILP systems can formulate hypotheses in first-order logic*
  - Can learn in environments not understood by simpler systems

# Explanation-based Learning

- Extracting general rules from individual observations
- *Example*: differentiating and simplifying algebraic expressions
  - Differentiate $X^2$ with respect to $X$ to get $2X$
  - Logical reasoning system

    Ask($Derivative(X^2, X) = d$, $KB$) with solution $d = 2X$
  - Solving this for the first time using standard rules of differentiation gives $1 \times (2 \times (X^{(2-1)}))$
  - Takes a first-time program 136 proof steps with 99 dead end branches
- Memoization
  - Speed up by saving the results of computation
  - Create a database of input/output pairs

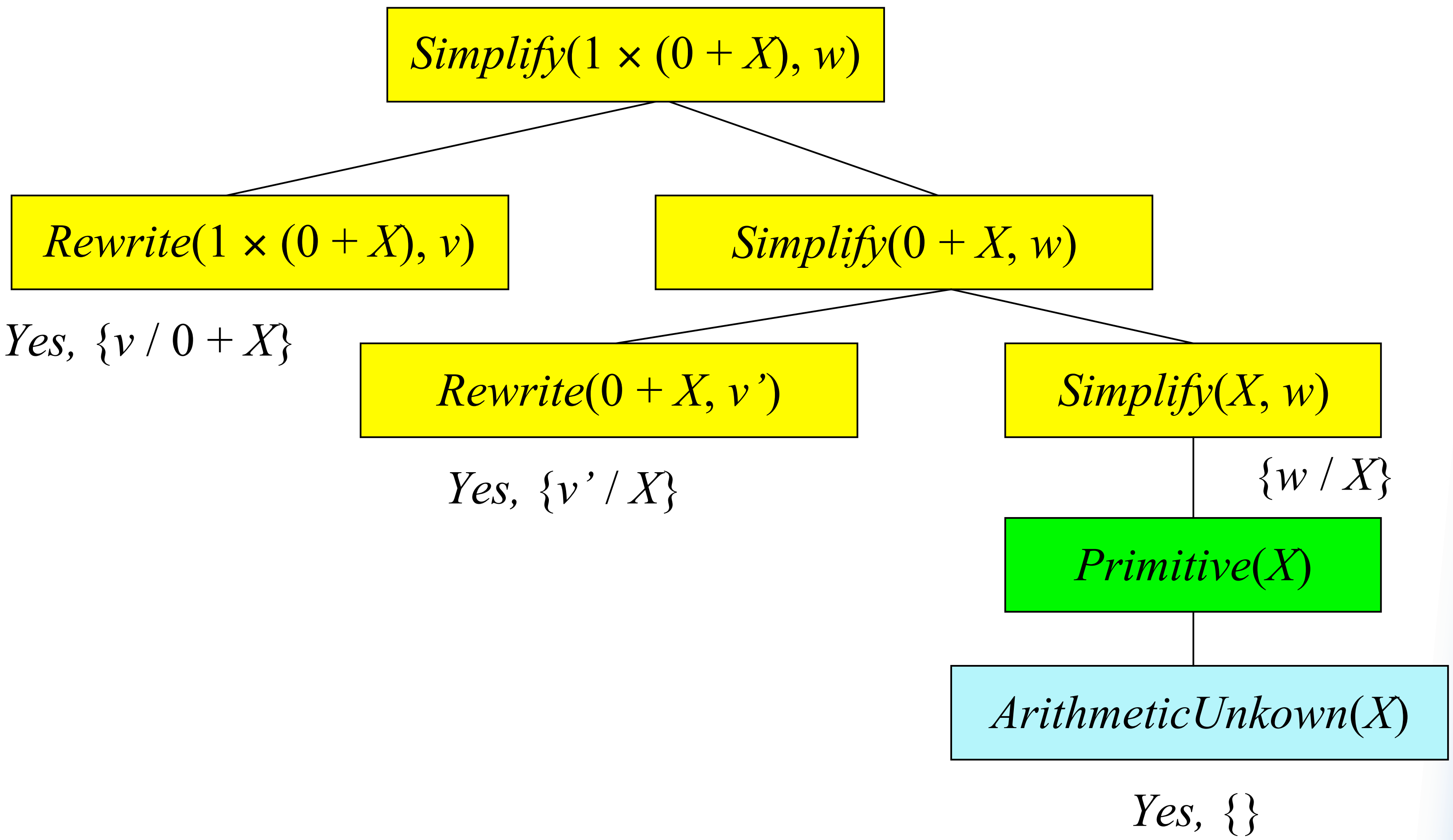
# Creating general rules

- Memoization in explanation-based learning
  - Create *general rules* that cover an entire class of cases
  - *Example*: extract the general rule
    $ArithmeticUnknown(u) \Rightarrow Derivative(u^2, u) = 2u$
- *Once something is understood, it can be generalized and reused in other circumstances*
  - "Civilization advances by extending the number of important operations that we can do without thinking about them"
- *Explaining why something is a good idea is much easier than coming up with the idea in the first place*
  - Watch caveman Zog roast his lizard vs. thinking about putting the lizard on a stick

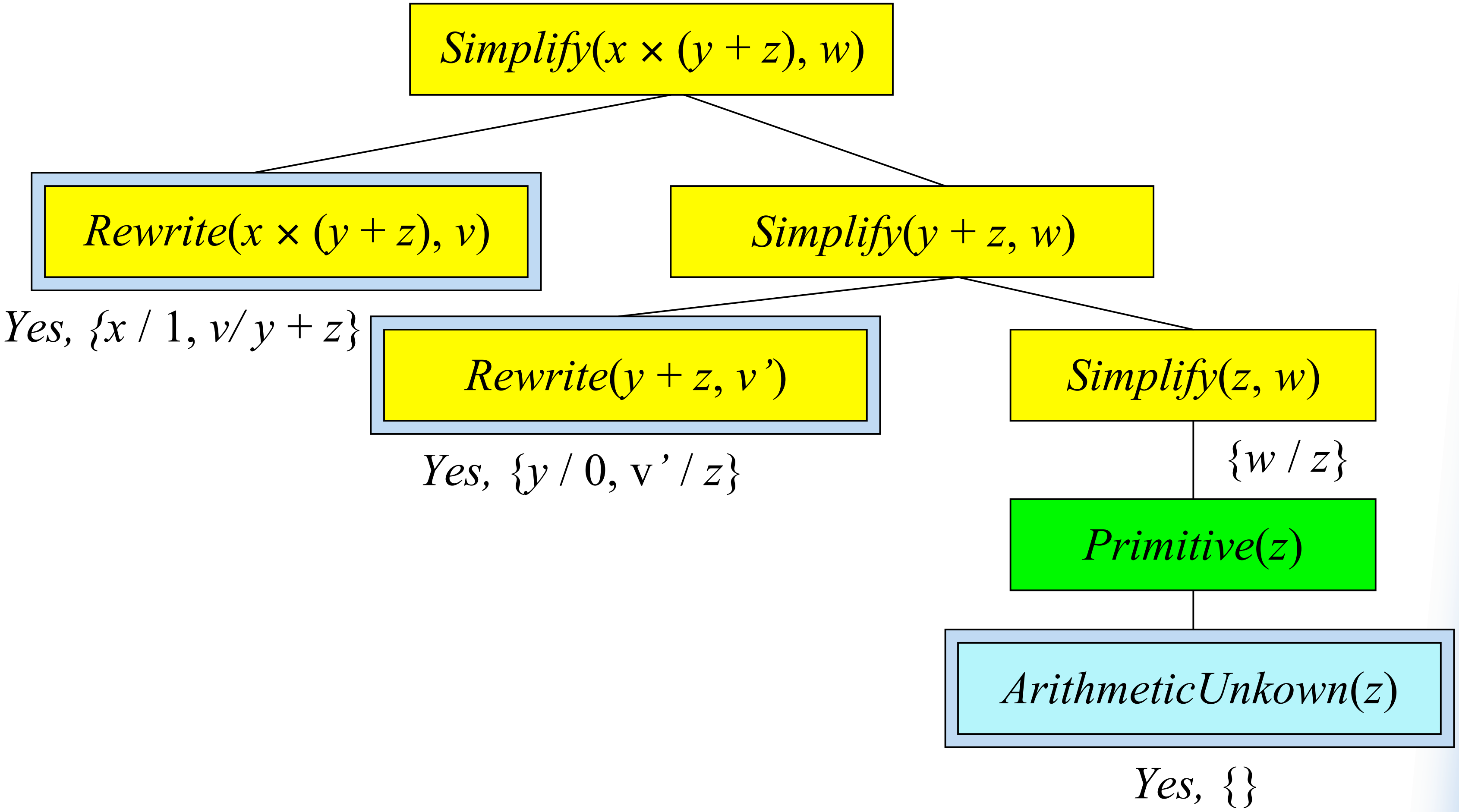
# Extracting rules from examples

- Basic idea behind EBL
  - Construct an explanation of the observation using prior knowledge
  - Establish a definition of the class of cases for which the same explanation can be used
- *Example*: simplifying $1 \times (0 + X)$ using a knowledge base with the following rules
  - $Rewrite(u, v) \wedge Simplify(v, w) \Rightarrow Simplify(u, w)$
  - $Primitive(u) \Rightarrow Simplify(u, u)$
  - $ArithmeticUnknown(u) \Rightarrow Primitive(u)$
  - $Number(u) \Rightarrow Primitive(u)$
  - $Rewrite(1 \times u, u)$
  - $Rewrite(0 + u, u)$
  - …

# Proof tree for original problem



# Generalized proof tree



# Generalizing proofs

- *The variabilized proof proceeds using exactly the same rule applications*
  - May lead to variable instantiation
- *Take the leaves of the generalized proof tree to get the general rule*

  $Rewrite(1 \times (0 + z), 0 + z) \wedge Rewrite(0 + z, z) \wedge ArithmeticUnknown(z) \Rightarrow Simplify(1 \times (0 + z), z)$
  - The first two conditions are independent of *z*, so this becomes

    $ArithmeticUnknown(z) \Rightarrow Simplify(1 \times (0 + z), z)$
- Recap
  - Use background knowledge to construct a proof for the example
  - In parallel, construct a generalized proof tree
  - New rule is the conjunction of the leaves of the proof tree and the variabilized goal
  - Drop conditions that are true regardless of the variables in the goal

# Improving efficiency

- Pruning the proof tree to get more general rules

  $Primitive(z) \Rightarrow Simplify(1 \times (0 + z), z)$

  $Simplify(y + z, w) \Rightarrow Simplify(1 \times (y + z), w)$

- *Problem*: Which rules to choose?
  - Adding large numbers of rules to the knowledge base slows down the reasoning process (increases the *branching factor* of the search space)
  - To compensate, the derived rules must offer significant speed increases
  - Derived rules should be as general as possible to apply to the largest possible set of cases

# Improving efficiency

- Operationality of subgoals in the rule
  - A subgoal must be "easy" to solve
  - *Primitive*($z$) is easy to solve, but *Simplify*($y + z, w$) leads to an arbitrary amount of inference
  - Keep operational subgoals and prune the rest of the tree
- Trade-off between operationality and generality
  - More specific subgoals are easier to solve but cover fewer cases
  - How many steps are still called operational?
  - Cost of a subgoal depends on the rules in the knowledge base

Maximizing the efficiency of an initial knowledge base is a complex optimization problem

# Improving efficiency

- Empirical analysis of efficiency
  - Average-case complexity on a population of problems that needs to be solved
- *By generalizing from past example problems, EBL makes the knowledge base more efficient for the kind of problems that it is reasonable to expect*
  - Works if the distribution of past problems is roughly the same as for future problems
  - Can lead to great improvement
    - Swedish to English translator was made 1200 times faster by using EBL

# Relevance-based Learning

- Functional dependencies or determinations
  - Background knowledge in Brazil example

$$\forall_{x,y,n,l} Nationality(x,n) \wedge Nationality(y,n) \wedge Language(x,l) \Rightarrow Language(y,l)$$

  - Therefore, from

$$Nationality(Fernando, Brazil) \wedge Language(Fernando, Portuguese)$$

    it follows

$$\forall_{x} Nationality(x, Brazil) \Rightarrow Language(x, Portuguese)$$

- Special syntax

$$Nationality(x,n) \succ Language(x,l)$$

# Determining the hypothesis space

- Determinations limit the hypothesis space
  - No possible conclusions about all nationalities from a single example
  - Only consider the important features (i.e. not day of the week, hair style of David Beckham)
- *Determinations specify a sufficient basis vocabulary from which to construct hypotheses*
- Reduction of the hypothesis space makes it easier to learn the target predicate
  - For Boolean functions log(|**H**|) examples are needed in a |**H**| size hypothesis space
  - Without restrictions, this is $O(2^n)$ examples
  - If the determination contains $d$ predicates on the left, only $O(2^d)$ examples are needed
  - Reduction of size $O(2^{n-d})$

# Learning relevance information

- Prior knowledge also needs to be learned
- Learning algorithm for determinations
  - Find the simplest determination consistent with the observations
  - A determination $P \succ Q$ says that if examples match *P* they must also match *Q*
  - *A determination is consistent with a set of examples if every pair that matches on the predicates on the left-hand side also matches on the target predicate*

# Learning relevance information

| Sample | Mass | Temp | Material | Size | Conductance |
|---|---|---|---|---|---|
| S1 | 12 | 26 | Copper | 3 | 0.59 |
| S1 | 12 | 100 | Copper | 3 | 0.57 |
| S2 | 24 | 26 | Copper | 6 | 0.59 |
| S3 | 12 | 26 | Lead | 2 | 0.05 |
| S3 | 12 | 100 | Lead | 2 | 0.04 |
| S4 | 24 | 26 | Lead | 4 | 0.05 |

- *Minimal consistent determination*

$$Material \wedge Temperature \succ Conductance$$

- *Non-minimal consistent determination*

$$Mass \wedge Size \wedge Temperature \succ Conductance$$

# Learning relevance information

```
function Minimimal-Consistent-Det(E, A) returns a determination
    inputs: E, a set of examples
            A, a set of attributes, of size n
    for i ← 0, …, n do
        for each subset A_i of A of size i do
            if Consistent-Det?(A_i ,E) then return A_i
        end
end

function Consistent-Det?(A, E) returns a truth-value
    inputs: A, a set of attributes
            E, a set of examples
    local variables: H, a hash table
    for each example e in E do
        if some example in H has the same value as e for the attributes A but a
    different classification then return False
        store the class of e in H, indexed by the values for attributes A of the
    example e
    end
    return True
```

# Complexity

- Time complexity depends on the size of the minimal consistent determination
  - In case of *p* attributes and a total of *n* attributes, the algorithm has to search all subsets of *A* of size *p*
  - *There are $O(n^p)$ of these, so the algorithm is exponential*
  - *The general problem is NP-complete*
  - In most domains there is sufficient local structure to make *p* small

# Relevance-based Decision Tree Learning

**function** RBDTL(*E, A, v*) **returns** a decision tree
  **return** DTL(*E*, Minimal-Consistent-Det(*E,A*), *v*)

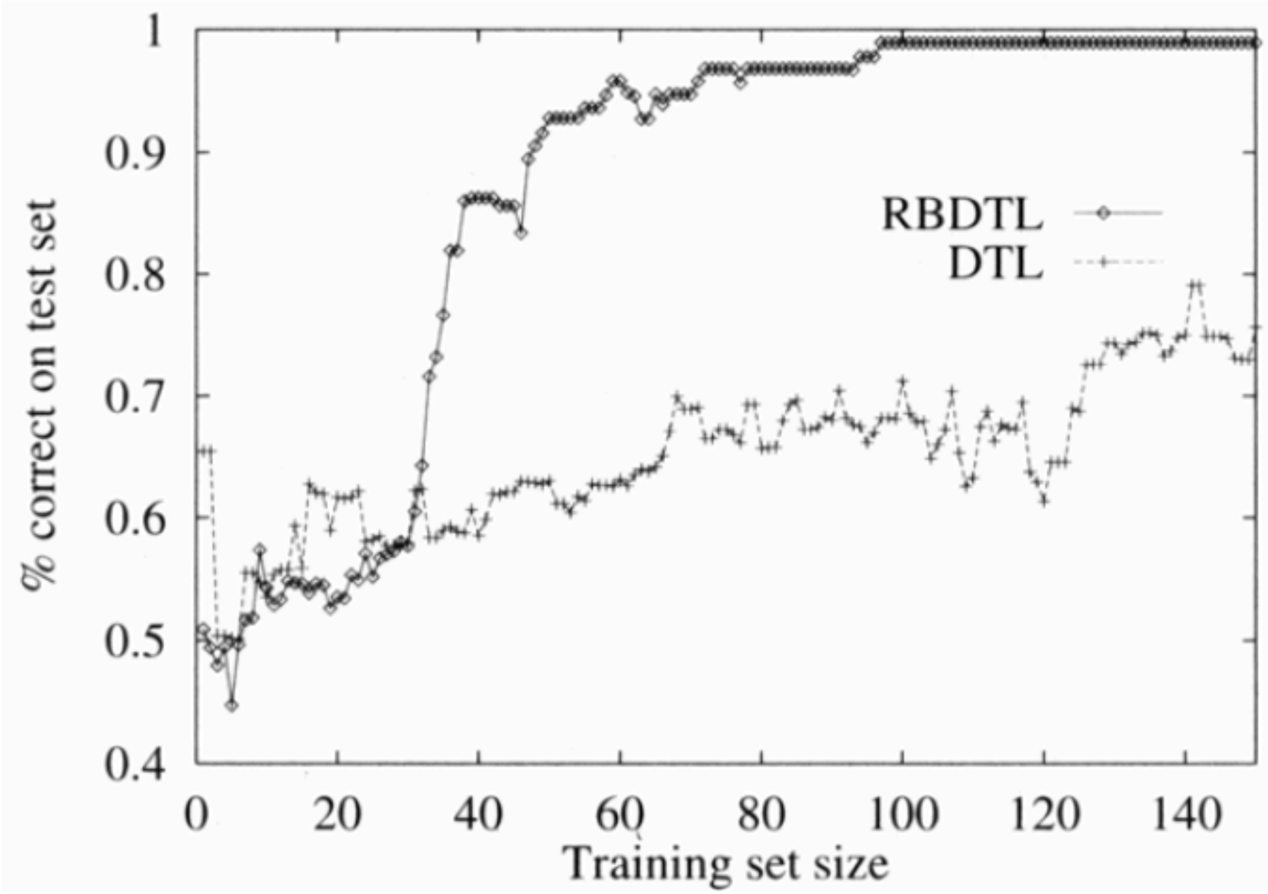
**function** DTL(*examples, attributes, default*) **returns** a decision tree
  **if** *examples* is empty **then return** *default*
  **else if** all *examples* have the same classification **then return** the classification
  **else if** *attributes* is empty **then return** MODE(*examples*)
  **else**
    *best* ← CHOOSE-ATTRIBUTE(*attributes, examples*)
    *tree* ← a new decision tree with root test *best*
    **for each** value $v_i$ of *best* **do**
      $examples_i$ ← {elements of *examples* with *best* $= v_i$}
      *subtree* ← DTL($examples_i$, *attributes* − *best*, MODE(*examples*))
      add a branch to *tree* with label $v_i$ and subtree *subtree*
    **return** *tree*

# Exploiting Knowledge

- RBDTL simultaneously learns and uses relevance information to minimize its hypothesis space

- Declarative bias
  - How can prior knowledge be used to identify the appropriate hypothesis space to search for the correct target definition?
  - *Unanswered questions*
    - How to handle noise?
    - How to use other kinds of prior knowledge besides determinations?
    - How can the algorithms be generalized to cover any first-order theory?

# RBDTL vs. DTL



RBDTL
DTL
% correct on test set
Training set size
1
0.9
0.8
0.7
0.6
0.5
0.4
0
20
40
60
80
100
120
140

# Inductive Logic Programming

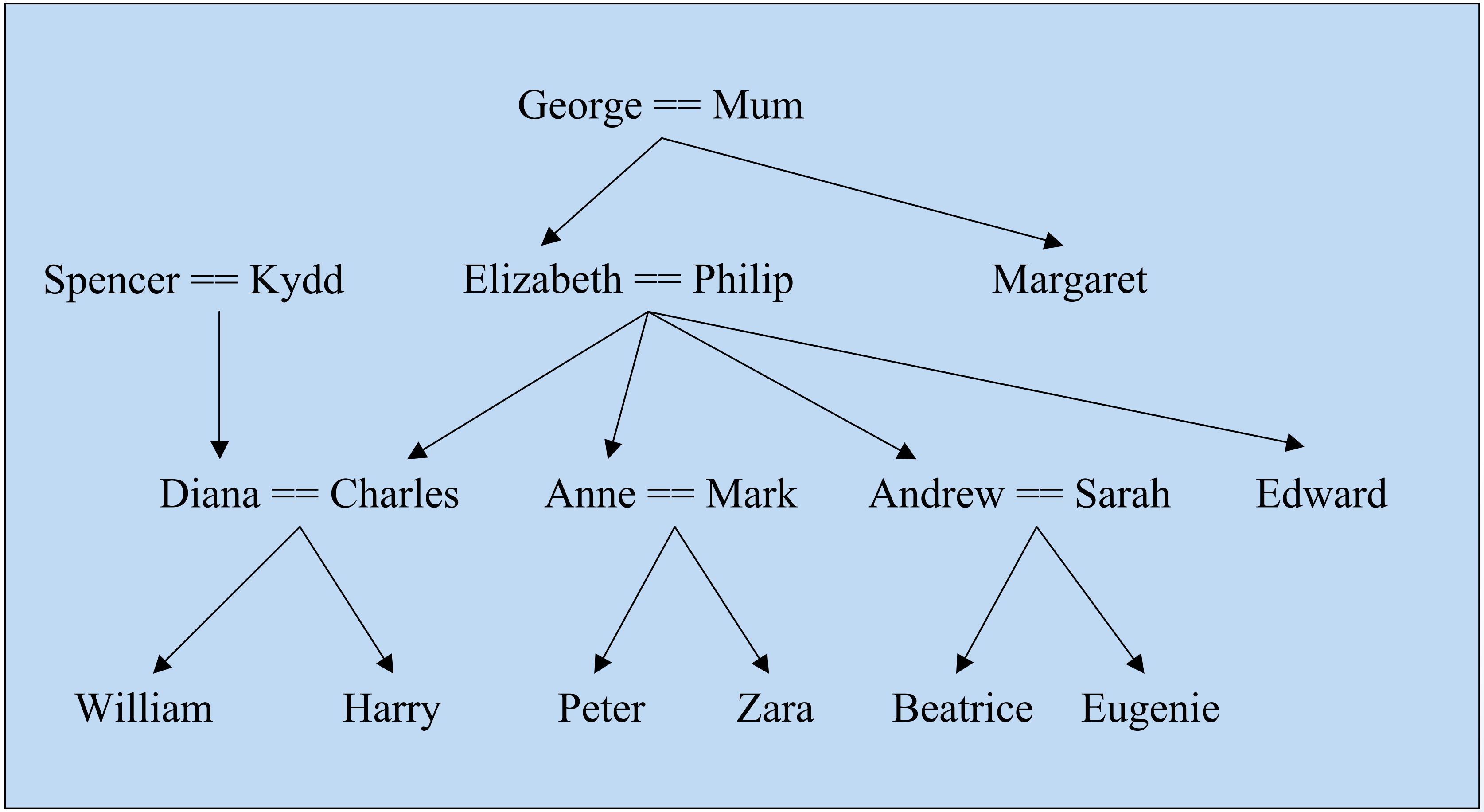
- Combines inductive methods with the power of first-order representations
- Offers a rigorous approach to the general KBIL problem
- Offers complete algorithms for inducing general, first-order theories from examples

# ILP: An example

- General knowledge-based induction problem

  $Background \wedge Hypothesis \wedge Descriptions \models Classifications$
- *Example*: Learning family relations from examples
  - Observations are an extended family tree
    - *Mother, Father* and *Married* relations
    - *Male* and *Female* properties
  - Target predicates: *Grandparent, BrotherInLaw, Ancestor*

# Example


George == Mum
Spencer == Kydd
Elizabeth == Philip
Margaret
Diana == Charles
Anne == Mark
Andrew == Sarah
Edward
William
Harry
Peter
Zara
Beatrice
Eugenie

# Example

- *Descriptions* include facts like
  - *Father(Philip, Charles)*
  - *Mother(Mum, Margaret)*
  - *Married(Diana, Charles)*
  - *Male(Philip)*
  - *Female(Beatrice)*
- Sentences in *Qualifications* depend on the target concept
  - *Grandparent(Mum, Charles)*
  - ¬*Grandparent(Mum, Harry)*
- *Goal*: find a set of sentences for *Hypothesis* such that the entailment constraint is satisfied
  - Without background knowledge this is for example

$$
\begin{aligned}
Grandparent(x,y) \Leftrightarrow & \left[\exists_z Mother(x,z) \wedge Mother(z,y)\right] \\
& \vee \left[\exists_z Mother(x,z) \wedge Father(z,y)\right] \\
& \vee \left[\exists_z Father(x,z) \wedge Mother(z,y)\right] \\
& \vee \left[\exists_z Father(x,z) \wedge Father(z,y)\right]
\end{aligned}
$$

# Why Attribute-based Learning Fails

- Decision-Tree-Learning will get nowhere
  - To express *Grandparent* as an attribute, pairs of people need to be objects
    *Grandparent*(<*Mum,Charles*>)
  - But then the example descriptions can not be represented
    *FirstElementIsMotherOfElizabeth*(<*Mum,Charles*>)
  - A large disjunction of specific cases without any hope of generalization to new examples

⇒ Attribute-based learning algorithms are incapable of learning relational predicates

# Background knowledge

- <u>A little bit of background knowledge helps a lot</u>
  - Background knowledge contains
    $$Parent(x,y) \Leftrightarrow [Mother(x,y) \vee Father(x,y)]$$
  - *Grandparent* is now reduced to
    $$Grandparent(x,y) \Leftrightarrow [\exists_z Parent(x,z) \wedge Parent(z,y)]$$
- <u>Constructive induction algorithm</u>
  - Create new predicates to facilitate the expression of explanatory hypotheses
  - *Example*: introduce a predicate *Parent* to simplify the definitions of the target predicates

# Top-down learning methods

- Top-down learning method
  - *Decision-tree learning*: start from the observations and work backwards
    - Decision tree is gradually grown until it is consistent with the observations
  - *Top-down learning*: start from a general rule and specialize it

# FOIL

- Split positive and negative examples
  - Positive: *<George, Anne>, <Philip, Peter>, <Spencer, Harry>*
  - Negative: *<George, Elizabeth>, <Harry, Zara>, <Charles, Philip>*
- Construct a set of Horn clauses with *Grandfather(x,y)* as the head with the positive examples instances of the *Grandfather* relationship
  - Start with a clause with an empty body
    $\Rightarrow$ *Grandfather(x,y)*
  - All examples are now classified as positive, so specialize
    1) *Father(x,y)* $\Rightarrow$ *Grandfather(x,y)*
    2) *Parent(x,z)* $\Rightarrow$ *Grandfather(x,y)*
    3) *Father(x,z)* $\Rightarrow$ *Grandfather(x,y)*
  - The first one incorrectly classifies the positive examples
  - The second one is incorrect on a larger part of the negative examples
  - Prefer the third clause and specialize
    *Father(x,z)* $\wedge$ *Parent(z,y)* $\Rightarrow$ *Grandfather(x,y)*

# FOIL

**function** Foil(*examples, target*) **returns** a set of Horn clauses
  **inputs**: *examples*, set of examples
    *target*, a literal for the goal predicate
  **local variables**: *clauses,* set of clauses, initially empty
  **while** *examples* contains positive examples **do**
    *clause* ← New-Clause(*examples, target*)
    remove examples covered by *clause* from *examples*
    add *clause* to *clauses*
  **return** *clauses*

# FOIL

**function** New-Clause(*examples, target*) **returns** a Horn clause
    **local variables**:
        *clause*, a clause with *target* as head and an empty body
        *l*, a literal to be added to the clause
        *extended-examples*, a set of examples with values for new variables
    *extended-examples* ← *examples*
    **while** *extended-examples* contains negative examples **do**
        *l* ← Choose-Literal(New-Literals(*clause*), *extended-examples*)
        append *l* to the body of *clause*
        *extended-examples* ← set of examples created by applying Extend-Example to each example in *extended-examples*
    return *clause*

# FOIL

```
function Extend-Example(example, literal) returns
    if example satisfies literal
        then return the set of examples created
            by extending example with each
            possible constant value for each new
            variable in literal
    else return the empty set
```

# FOIL

- New-Literals
  - Takes a clause and constructs all possible "useful" literals
- *Example*: *Father(x,z)* $\Rightarrow$ *Grandfather(x,y)*
  - *Add literals using predicates*
    - Negated or unnegated
    - Use any existing predicate (including the goal)
    - Arguments must be variables
    - Each literal must include at least one variable from an earlier literal or from the head of the clause
    - Valid: *Mother(z,u), Married(z,z), Grandfather(v,x)*
    - Invalid: *Married(u.v)*
  - *Equality and inequality literals*
    - E.g. $z \neq x$, empty list
  - *Arithmetic comparisons*
    - E.g. $x > y$, threshold values

# FOIL

- The way New-Literal changes the clauses leads to a very large branching factor
- Improve performance by using type information
  - E.g., *Parent(x,n)* where *x* is a person and *n* is a number
- <u>Choose-Literal uses a heuristic similar to information gain</u>
- <u>Ockham's razor to eliminate hypotheses</u>
  - If the clause becomes longer than the total length of the positive examples that the clause explains, this clause is not a valid hypothesis
- *Most impressive demonstration*
  - Learn the correct definition of list-processing functions in Prolog from a small set of examples, using previously learned functions as background knowledge

# Inverse Resolution

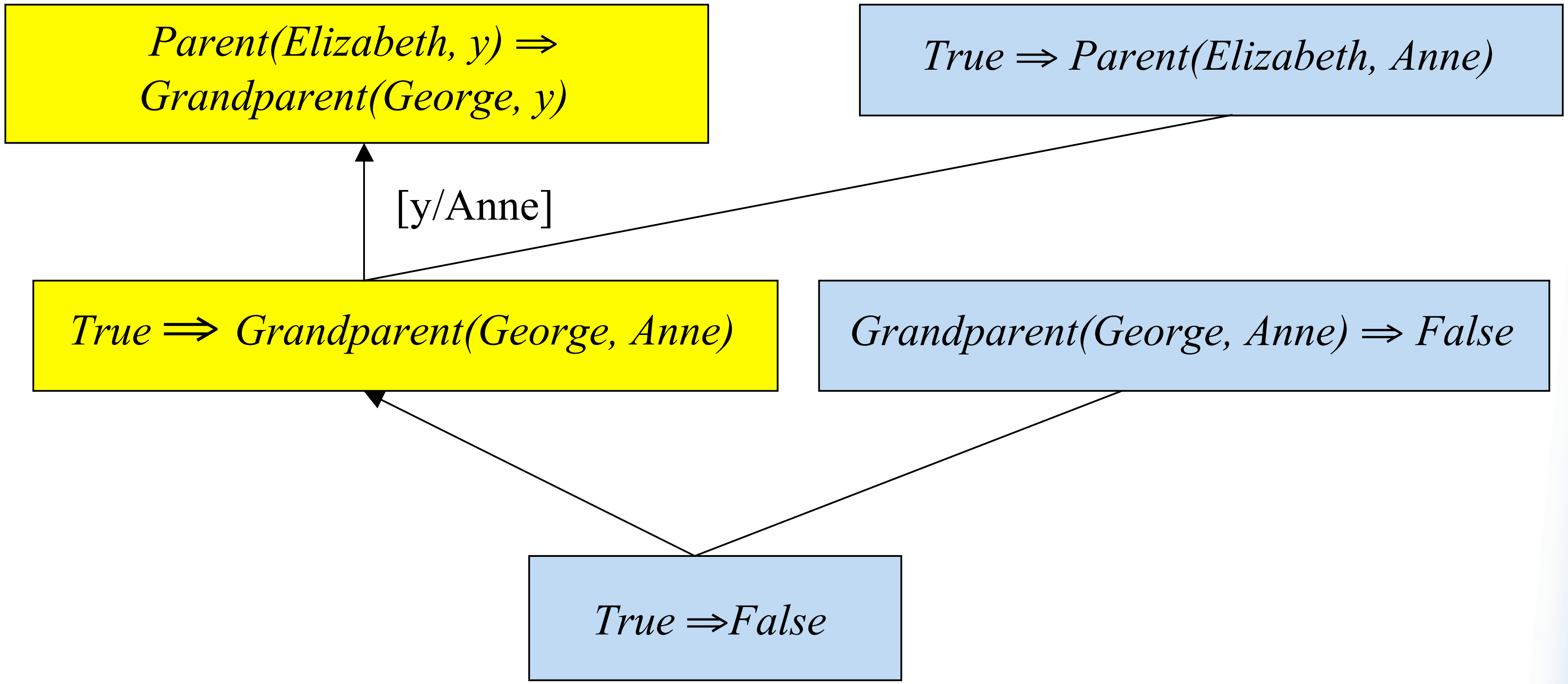
- Inverse resolution
  - *Classifications* follows from

    *Background* ∧ *Hypothesis* ∧ *Descriptions*
  - This can be proven by resolution
  - Run the proof backwards to find *Hypothesis*
  - *Problem*: How to run the proof backwards?

# Generating Inverse Proofs

- Ordinary resolution
  - Take two clauses $C_1$ and $C_2$ and resolve them to produce the *resolvent* $C$
- Inverse resolution
  - Take resolvent $C$ and produce two clauses $C_1$ and $C_2$
  - Take $C$ and $C_1$ and produce $C_2$

# Generating Inverse Proofs



# Generating Inverse Proofs

- Inverse resolution is a search
  - For any $C$ and $C_1$ there can be several or even an infinite number of clauses $C_2$
    - Instead of *Parent(Elizabeth,y)* $\Rightarrow$ *Grandparent(George,y)* there were numerous alternatives

      *Parent(Elizabeth,Anne)* $\Rightarrow$ *Grandparent(George,Anne)*

      *Parent(z,Anne)* $\Rightarrow$ *Grandparent(George,Anne)*

      *Parent(z,y)* $\Rightarrow$ *Grandparent(George,y)*
  - The clauses $C_1$ that participate in each step can be chosen from *Background, Descriptions, Classifications* or from hypothesized clauses already generated
- ILP needs restrictions to make the search manageable
  - Eliminate function symbols
  - Generate only the most specific hypotheses
  - Use Horn clauses
  - All hypothesized clauses must be consistent with each other
  - Each hypothesized clause must agree with the observations

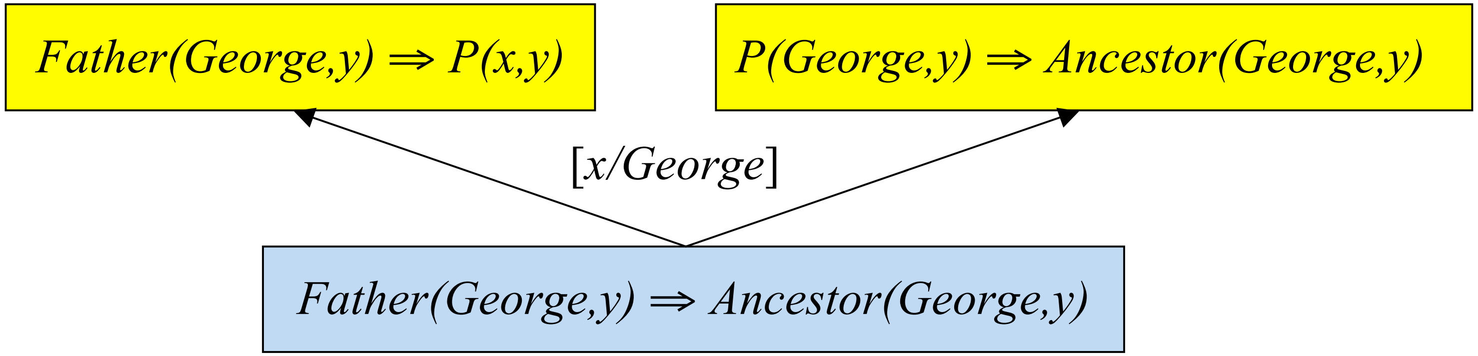
# New Predicates and New Knowledge

- An inverse resolution procedure is a complete algorithm for learning first-order theories
  - If some unknown *Hypothesis* generates a set of examples, then an inverse resolution procedure can generate *Hypothesis* from the examples
- Can inverse resolution infer the law of gravity from examples of falling bodies?
  - Yes, given suitable background mathematics
- *Monkey and typewriter problem*: How to overcome the large branching factor and the lack of structure in the search space?

# New Predicates and New Knowledge

- Inverse resolution is capable of generating new predicates
  - Resolution of $C_1$ and $C_2$ into $C$ eliminates a literal that $C_1$ and $C_2$ share
  - This literal might contain a predicate that does not appear in $C$
  - When working backwards, one possibility is to generate a new predicate from which to construct the missing literal

# New Predicates and New Knowledge




- *P* can be used in later inverse resolution steps
  - *Example*: $Mother(x,y) \Rightarrow P(x,y)$ or $Father(x,y) \Rightarrow P(x,y)$ leading to the "Parent" relationship
- <u>Inventing new predicates is important to reduce the size of the definition of the goal predicate</u>
  - Some of the deepest revolutions in science come from the invention of new predicates (e.g. Galileo's invention of acceleration)

# Applications

- ILP systems have outperformed knowledge-free methods in a number of domains
  - *Molecular biology*: the GOLEM system has been able to generate high-quality predictions of protein structures and the therapeutic efficacy of various drugs
  - GOLEM is a completely general-purpose program that is able to make use of background knowledge about any domain

# Knowledge in Learning: Summary

- Cumulative learning
  - Improve learning ability as new knowledge is acquired
- *Prior knowledge helps to eliminate hypothesis and fills in explanations, leading to shorter hypotheses*
- Entailment constraints
  - Logical definition of different learning types
- Explanation-based learning (EBL)
  - Explain the examples and generalize the explanation
- Relevance-base learning (RBL)
  - Use prior knowledge in the form of determinations to identify the relevant attributes
- Knowledge-based inductive learning (KBIL)
  - Finds inductive hypotheses that explain sets of observations
- Inductive logic programming (ILP)
  - Perform KBIL using knowledge expressed in first-order logic
  - Generates new predicates with which concise new theories can be expressed