12

AC Voltage Regulators

AC voltage regulators have a constant voltage ac supply input and incorporate semiconductor switches which vary the rms voltage impressed across the ac load. These regulators fall into the category of naturally commutating converters since their thyristor switches are commutated by the alternating supply. This converter turn-off process is termed *line commutation*.
The regulator output current, hence supply current, may be discontinuous or non-sinusoidal and as a consequence input power factor correction and harmonic reduction are usually necessary, particularly at low output voltage levels.
A feature of direction conversion of ac to ac is the absence of any intermediate energy stage, such as a dc link. Therefore ac to ac converters are potentially more efficient but usually involve a larger number of switching devices and output is lost if the input supply is temporarily lost.
There are three basic ac regulator categories, depending on the relationship between the input supply frequency f_s, which is usually assume single frequency sinusoidal, and the output frequency f_o. Without the use of transformers, the output voltage rms magnitude V_{Orms} is less than or equal to the input voltage rms magnitude V_s , $V_{Orms} \le V_s$.

- output frequency increased, $f_o > f_s$
- output frequency decreased, $f_o < f_s$
- output frequency fundamental = supply frequency, $f_o = f_s$

12.1 Single-phase ac regulator

Figure 12.la shows a single-phase thyristor regulator supplying an *L-R* load. The two thyristors can be replaced by any of the bidirectional conducting and blocking switch arrangements shown in figure 6.11. Equally, in low power applications the two thyristors are usually replaced by a triac. The thyristor gate trigger delay angle is α, as indicated in figure 12.lb. The fundamental of the output frequency is the same as the input frequency, $\omega = 2\pi f_s$. The thyristor current, shown in figure 12.lb, is defined by equation (11.33); that is

$$L\frac{di}{dt} + Ri = \sqrt{2}V \sin \omega t \quad \text{(V)} \qquad \alpha \le \omega t \le \beta \quad \text{(rad)} \qquad (12.1)$$
$$= 0 \qquad \text{otherwise}$$

The solution to this first order differential equation has two solutions, depending on the delay angle α relative to the load natural power factor angle, $\phi = \tan^{-1} \omega L/R$.
Because of symmetry around the time axis, the mean supply and load, voltages and currents, are zero.

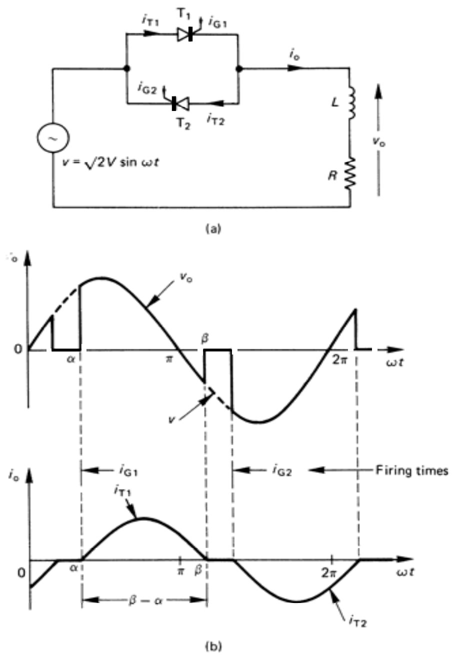

Figure 12.1. *Single-phase full-wave thyristor ac regulator with an R-L load: (a) circuit connection and (b) load current and voltage waveforms.*

12.1i - Case 1: $\alpha > \phi$

When the delay angle exceeds the power factor angle the load current always reaches zero, thus the differential equation boundary conditions are zero. The solution for i is

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\left\{\sin\ (\omega t - \phi) - \sin(\alpha - \phi)\ e^{-\omega t + \alpha / \tan\phi}\right\} \quad \text{(A)} \qquad \alpha \le \omega t \le \beta \quad \text{(rad)} \tag{12.2}$$

$$i(\omega t) = 0 \quad \text{(A)} \qquad \pi \le \beta \le \omega t \le \pi + \alpha \quad \text{(rad)} \tag{12.3}$$

where $Z = \sqrt{(R^2 + \omega^2 L^2)}$ (ohms) and $\tan\phi = \omega L / R$

Provided $\alpha > \phi$ both regulator thyristors will conduct and load current flows symmetrically as shown in figure 12.lb.

The thyristor current extinction angle β for discontinuous load current can be determined with the aid of figure 11.7a, but with the restriction that $\beta - \alpha \le \pi$ or by solving equation 11.39, that is:

$$\sin(\beta - \phi) = \sin(\alpha - \phi)\ e^{(\alpha-\beta)/\tan\phi}$$

From figure 12.lb the rms output voltage is

$$V_{rms} = \left[{}^1\!/\!_\pi \int_\alpha^\beta \left(\sqrt{2}\,V\right)^2 \sin^2 \omega t \ d\omega t \right]^{½} = \sqrt{2}\,V \left[{}^1\!/\!_{2\pi} \int_\alpha^\beta \left(1 - \cos 2\omega t\right) d\omega t \right]^{½}$$
$$= V \left[{}^1\!/\!_\pi \left\{ \left(\beta - \alpha\right) - ½(\sin 2\beta - \sin 2\alpha) \right\} \right]^{½} \tag{12.4}$$

12.1ii - Case 2: $\alpha \le \phi$

When $\alpha \le \phi$, a pure sinusoidal load current flows, and substitution of $\alpha = \phi$ in equation (12.2) results in

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\sin\ (\omega t - \phi) \quad \text{(A)} \qquad \alpha \le \phi \quad \text{(rad)} \tag{12.5}$$

The rms output voltage is V, the sinusoidal supply voltage rms value.

The power delivered to the load is therefore

$$P_o = I_{rms}^2 R = \frac{V^2}{Z}\cos\phi \tag{12.6}$$

If a short duration gate trigger pulse is used and $\alpha < \phi$, unidirectional load current will result. The device to be turned on is reverse-biased by the conducting device. Thus if the gate pulse ceases before the load current has fallen to zero, only one device conducts. It is therefore usual to employ a continuous gate pulse, or stream of pulses, from α until π, then for $\alpha < \phi$ a sine wave output current results.

In both load angle cases, the following equations are valid, except $\beta = \pi + \alpha$ is used for case 2, when $\alpha \le \phi$.

The rectified mean voltage can be used to determine the thyristor mean current rating.

$$\overline{V}_o = {}^1\!/\!_\pi \int_\alpha^\beta \sqrt{2}\,V \sin \omega t \ d\omega t$$
$$= \sqrt{2}\,V \left[{}^1\!/\!_\pi \left\{ \cos\alpha - \cos\beta \right\} \right] \quad \text{(V)} \tag{12.7}$$

The mean thyristor current $\overline{I}_{Th} = ½\overline{I}_o = ½\overline{V}_o / R$, that is

$$\overline{I}_{Th} = \frac{½\overline{V}_o}{R} = \frac{\sqrt{2}\,V}{2R}\left[{}^1\!/\!_\pi \left\{ \cos\alpha - \cos\beta \right\} \right] \quad \text{(A)} \tag{12.8}$$

The maximum mean thyristor current is for a resistive load, $\alpha = 0$, and $\beta = \pi$, that is

$$\hat{\overline{I}}_{Th} = \sqrt{2}V \big/ \pi R \tag{12.9}$$

The rms load current is found by the appropriate integration of equation (12.2), namely

$$I_{rms} = \left[\tfrac{1}{\pi} \int_\alpha^\beta \left(\frac{\sqrt{2}V}{Z} \right)^2 \left\{ \sin\ (\omega t - \phi) - \sin(\alpha - \phi)\ e^{-\omega t + \alpha / \tan\phi} \right\}^2 d\omega t \right]^{½}$$
$$= \frac{V}{Z}\left[\tfrac{1}{\pi} \left(\beta - \alpha - \frac{\sin(\beta - \alpha)}{\cos\phi} \cos(\beta + \alpha + \phi) \right) \right]^{½} \tag{12.10}$$

The thyristor maximum rms current is given by $I_{Th_{rms}} = I_{Orms} / \sqrt{2}$ when $\alpha \le \phi$, that is

$$\hat{I}_{Th\ rms} = V \big/ \sqrt{2}Z \tag{12.11}$$

The thyristor forward and reverse voltage blocking ratings are both $\sqrt{2}V$.

The fundamental load voltage components are

$$a_1 = \frac{\sqrt{2}\,V}{2\pi}\left\{ \cos 2\alpha - \cos 2\beta \right\}$$
$$b_1 = \frac{\sqrt{2}\,V}{2\pi}\left\{ 2\left(\beta - \alpha\right) - \sin 2\beta - \sin 2\alpha \right\} \tag{12.12}$$

If $\alpha \le \phi$, then continuous load current flows, and equation (12.12) reduces to $a_1 = 0$ and $b_1 = \sqrt{2}V$, when $\beta = \alpha + \pi$ is substituted.

12.1.1 Resistive Load

For a purely resistive load, the load voltage and current are related according to

$$i_o\left(\omega t\right) = v_o\left(\omega t\right) \big/ R = \sqrt{2}V \sin\left(\omega t\right) \big/ R \qquad \alpha \le \omega t \le \pi, \quad \alpha + \pi \le \omega t \le 2\pi$$
$$= 0 \qquad \text{otherwise}$$

The equations (12.1) to (12.6) can be simplified if the load is purely resistive. Continuous output current only flows for $\alpha = 0$, since $\phi = \tan^{-1} 0 = 0°$. Therefore the output equations are derived from the discontinuous equations (12.2) to (12.4).

The mean half-cycle output voltage, used to determine the thyristor mean current rating, is found by integrating the supply voltage over the interval α to π, ($\beta = \pi$).

$$V_o = \tfrac{1}{\pi}\int_\alpha^\pi \sqrt{2}\,V \sin\omega t \; d\omega t$$

$$= \tfrac{\sqrt{2}V}{\pi}(1+\cos\alpha) \qquad \text{(V)}$$

$$\overline{I}_o = V_o / R = \tfrac{\sqrt{2}V}{\pi}(1+\cos\alpha) \qquad \text{(A)}$$

$$\overline{I}_{Th} = \tfrac{1}{2}\overline{I}_o$$

From equation (12.4) the rms output voltage for a delay angle α is

$$V_{rms} = \sqrt{\tfrac{1}{\pi}\int_\alpha^\pi \left(\sqrt{2}\,V \sin\omega t\right)^2 d\omega t}$$

$$= \sqrt{2}\,V\sqrt{\frac{2(\pi-\alpha)+\sin 2\alpha}{4\pi}} \qquad \text{(V)} \qquad (12.13)$$

Therefore the output power is

$$P_o = \frac{V_{rms}^2}{R} = \frac{V^2}{R}\left\{1-\frac{2\alpha-\sin 2\alpha}{2\pi}\right\} \qquad \text{(W)} \qquad (12.14)$$

The rms output current and supply current from $P_o = I_{rms}^2 R$ is

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V}{R}\sqrt{1-\frac{2\alpha-\sin 2\alpha}{2\pi}} \qquad \text{(A)}$$

and

$$I_{Trms} = I_{rms} / \sqrt{2}$$

The supply power factor λ is defined as the ratio of the real power to the apparent power, that is

$$\lambda = \frac{P_o}{S} = \frac{V_{rms} I}{V I} = \frac{V_{rms}}{V} = \sqrt{\frac{2(\pi-\alpha)+\sin 2\alpha}{4\pi}}$$

Example 12.1a: ***single-phase ac regulator - 1***

If the load of the 50 Hz 240V ac voltage regulator shown in figure 12.1 is Z = 7.1+j7.1 Ω, calculate the load natural power factor angle, ϕ. Then calculate

(a) the rms output voltage, and hence

(b) the output power and rms current, whence input power factor

for

i. $\alpha = \tfrac{1}{6}\pi$

ii. $\alpha = \tfrac{1}{3}\pi$

Solution

From equation (12.3) the load natural power factor angle is

$$\phi = \tan^{-1}\omega L / R = \tan^{-1} X_L / R$$

$$= \tan^{-1} 7.1/7.1 = \tfrac{1}{4}\pi \text{ (rad)}$$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{7.1^2 + 7.1^2} = 10\Omega$$

i. $\alpha = \tfrac{1}{6}\pi$

(a) Since $\alpha = \pi/6 < \phi = \pi/4$, the load current is continuous. The rms load voltage is 240V.

(b) From equation (12.6) the power delivered to the load is

$$P_o = I_{rms}^2 R = \frac{V^2}{Z}\cos\phi$$

$$= \frac{240^2}{10}\cos\tfrac{1}{4}\pi = 4.07\text{kW}$$

The rms output current and supply current are both given by

$$I_{rms} = \sqrt{P_o / R}$$

$$= \sqrt{4.07\text{kW} / 7.1\Omega} = 23.8\text{A}$$

The input power factor is the load natural power factor, that is

$$pf = \frac{P_o}{S} = \frac{4.07\text{kW}}{240\text{V}\times 23.8\text{A}} = 0.70$$

ii. $\alpha = \tfrac{1}{3}\pi$

(a) Since $\alpha = \pi/3 > \phi = \tfrac{1}{4}\pi$, the load hence supply current is discontinuous. For $\alpha = \pi/3 > \phi = \tfrac{1}{4}\pi$ the extinction angle $\beta = \pi$ can be extracted from figure 11.7a. The rms load voltage is given by equation (12.4).

$$V_{rms} = V\left[\tfrac{1}{\pi}\left\{(\pi-\alpha) - \tfrac{1}{2}(\sin 2\pi - \sin 2\alpha)\right\}\right]^{\frac{1}{2}}$$

$$= 240\times\left[\tfrac{1}{\pi}\left\{(\pi-\tfrac{1}{3}\pi) - \tfrac{1}{2}(\sin 2\pi - \sin\tfrac{2}{3}\pi)\right\}\right]^{\frac{1}{2}}$$

$$= 240\times\sqrt{\tfrac{11}{12}} = 229.8\text{V}$$

The rms output current is given by equation (12.10), that is

$$I_{Orms} = \frac{V}{Z}\left[\tfrac{1}{\pi}\left(\beta-\alpha-\frac{\sin(\beta-\alpha)}{\cos\phi}\cos(\beta+\alpha+\phi)\right)\right]^{\frac{1}{2}}$$

$$= \frac{240}{10}\left[\tfrac{1}{\pi}\left(\pi-\tfrac{1}{3}\pi-\frac{\sin(\pi-\tfrac{1}{3}\pi)}{\cos\tfrac{1}{4}\pi}\cos(\pi+\tfrac{1}{3}\pi+\tfrac{1}{4}\pi)\right)\right]^{\frac{1}{2}}$$

$$= 18.1\text{A}$$

The output power is given by

$$P_o = I_{rms}^2 R$$
$$= 18.1^2 \times 7.1\Omega = 2313\text{W}$$

The load power factor is

$$pf = \frac{P_o}{S} = \frac{2313\text{W}}{229.8\text{V}\times 18.1\text{A}} = 0.56$$

♣

Example 12.1b: ***single-phase ac regulator - 2***

If the load of the 50 Hz 240V ac voltage regulator shown in figure 12.1 is $Z = 7.1+j7.1\ \Omega$, calculate the minimum controllable delay angle. Using this angle calculate

i. maximum rms output voltage and current, and hence
ii. maximum output power and power factor
iii. thyristor *I-V* and *di/dt* ratings

Solution

As in example 12.1a, from equation (12.3) the load natural power factor angle is

$$\phi = \tan^{-1}\omega L/R = \tan^{-1} 7.1/7.1 = \pi/4$$

The load impedance is Z=10Ω. The controllable delay angle range is $\tfrac{1}{4}\pi \le \alpha \le \pi$.

i. The maximum controllable output occurs when $\alpha = \tfrac{1}{4}\pi$.

From equation (12.2) when $\alpha = \phi$ the output voltage is the supply voltage, V, and

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\sin\ (\omega t - \tfrac{1}{4}\pi) \qquad \text{(A)}$$

The load hence supply rms maximum current, is therefore

$$I_{rms} = 240\text{V}/10\Omega = 24\text{A}$$

ii. Power $= I_{rms}^2 R = 24^2\times 7.1\Omega = 4090\text{W}$

$$\text{power factor} = \frac{\text{power output}}{\text{apparent power output}}$$
$$= \frac{I_{rms}^2 R}{V I_{rms}} = \frac{24^2\times 7.1\Omega}{240\text{V}\times 10\text{A}} = 0.71 \quad (=\cos\phi)$$

iii. Each thyristor conducts for π radians, between α and $\pi+\alpha$ for T1 and between $\pi+\alpha$ and $2\pi+\alpha$ for T2. The thyristor average current is

$$\bar{I}_T = \tfrac{1}{2\pi}\int_{\alpha=\phi}^{\alpha+\pi=\phi+\pi} \sqrt{2}\,V\sin(\omega t-\phi)\,d\omega t$$
$$= \frac{\sqrt{2}\,V}{\pi Z} = \frac{\sqrt{2}\times 240\text{V}}{\pi\times 10\Omega} = 10.8\text{A}$$

The thyristor rms current rating is

$$I_{Trms} = \left[\tfrac{1}{2\pi}\int_{\alpha=\phi}^{\alpha+\pi=\phi+\pi}\left\{\sqrt{2}\,V\sin(\omega t-\phi)\right\}^2 d\omega t\right]^{1/2}$$
$$= \frac{\sqrt{2}\,V}{2Z} = \frac{\sqrt{2}\times 240\text{V}}{2\times 10\Omega} = 17.0\text{A}$$

Maximum thyristor di/dt is derived from

$$d\,i(\omega t)/dt = d/dt\ \frac{\sqrt{2}V}{Z}\sin\ (\omega t - \tfrac{1}{4}\pi)$$
$$= \frac{\sqrt{2}V}{Z}\omega\cos\ (\omega t - \tfrac{1}{4}\pi) \qquad \text{(A/s)}$$

This has a maximum value when $\omega t - \tfrac{1}{4}\pi = 0$, that is at $\omega t = \alpha = \phi$, then

$$d\hat{i}(\omega t)/dt = \frac{\sqrt{2}V\omega}{Z}$$
$$= \frac{\sqrt{2}\times 240\text{V}\times 2\pi\times 50\text{Hz}}{10\Omega}$$
$$= 10.7\text{A/ms}$$

Thyristor forward and reverse blocking voltage requirements are $\sqrt{2}V = \sqrt{2}\times 240$.

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12.2 Three-phase ac regulator

12.2.1 Fully-controlled three-phase ac regulator

The power to a three-phase star or delta-connected load may be controlled by the ac regulator shown in figure 12.2a with a star-connected load shown. If a neutral connection is made, load current can flow provided at least one thyristor is conducting. At high power levels, neutral connection is to be avoided, because of load triplen currents that may flow through the phase inputs and the neutral. With a balanced delta connected load, no triplen or even harmonic currents occur.

If the regulator devices in figure 12.2a, without the neutral connected, were diodes, each would conduct for ½π in the order T_1 to T_6 at ⅓π radians apart.

In the fully controlled ac regulator of figure 12.2a without a neutral connection, at least two devices must conduct for power to be delivered to the load. The thyristor trigger sequence is as follows. If thyristor T_1 is triggered at α, then for a symmetrical three-phase load voltage, the other trigger angles are T_3 at $\alpha+\tfrac{2}{3}\pi$ and T_5 at $\alpha+4\pi/3$. For the antiparallel devices, T_4 (which is in antiparallel with T_1) is triggered at $\alpha+\pi$, T_6 at $\alpha+5\pi/3$, and finally T_2 at $\alpha+7\pi/3$.

Figure 12.2b shows resistive load, line-to-neutral voltage waveforms for four different phase delay angles, α. Three distinctive conduction periods (plus a non-conduction period) exist.

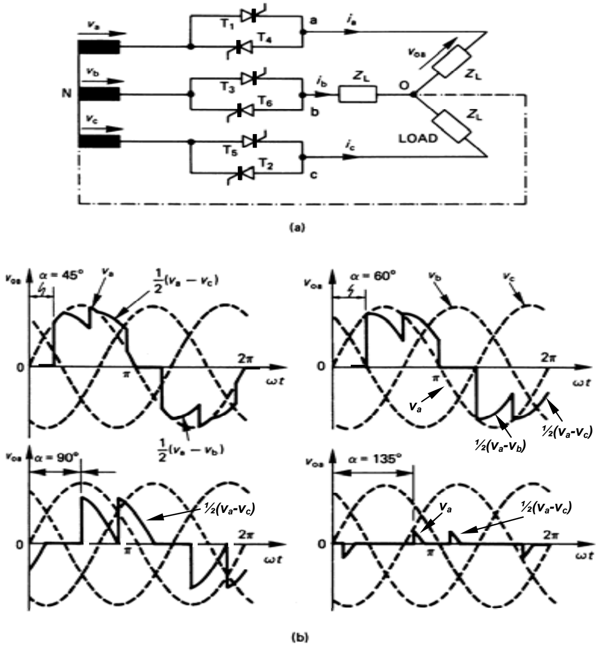

Figure 12.2. *Three-phase ac full-wave voltage controller: (a) circuit connection with a star load and (b) phase a, line-to-load neutral voltage waveforms for four firing delay angles.*

i. $0 \le \omega t \le \frac{1}{3}\pi$ [mode 2/3]

Full output occurs when $\alpha = 0$. For $\alpha \le \frac{1}{3}\pi$ three alternating devices conduct and one will be turned off by natural commutation. Only for $\omega t \le \frac{1}{3}\pi$ can three sequential devices be on simultaneously.

ii. $\frac{1}{3}\pi \le \omega t \le \frac{1}{2}\pi$ [mode 2/2]

The turning on of one device naturally commutates another conducting device and only two phases can be conducting, that is, only two thyristors conduct at any time. Line-to-neutral load voltage waveforms for $\alpha = \frac{1}{3}\pi$ and $\frac{1}{2}\pi$ are shown in figures 12.2b.

iii. $\frac{1}{2}\pi \le \omega t \le \frac{5}{6}\pi$ [mode 0/2]

Two devices must be triggered in order to establish load current and only two devices conduct at anytime. Line-to-neutral zero voltage periods occur and each device must be retriggered $\frac{1}{3}\pi$ after the initial trigger pulse. These zero output periods which develop for $\alpha \ge \frac{1}{2}\pi$ can be seen in figure 12.2b and are due to a previously on device commutating at $\omega t = \frac{5}{6}\pi$ then re-conducting at $\alpha + \frac{1}{3}\pi$. Except for regulator start up, the second firing pulse is not necessary if $\alpha \le \frac{1}{2}\pi$.

The interphase voltage falls to zero at $\alpha = \frac{5}{6}\pi$, hence for $\alpha \ge \frac{5}{6}\pi$ the output becomes zero.

Example 12.2: ***Three-phase ac regulator***

Evaluate expressions for the rms phase voltage ($V_{LL} = \sqrt{3}V_{phase} = \sqrt{3}\sqrt{2}\,V$) of the three-phase ac thyristor regulator shown in figure 12.2a, with a star-connected, balanced resistive load.

Solution

The waveforms in figure 12.2b are useful in determining the required bounds of integration. When three regulator thyristors conduct, the voltage (and the current) is of the form $\hat{V}/\sqrt{3}\,\sin\phi$, while when two devices conduct, the voltage (and the current) is of the form $\hat{V}/2\,\sin(\phi - \frac{1}{6}\pi)$. $\hat{V}$ is the maximum line voltage.

For phase delay angles $0 \le \alpha \le \frac{1}{3}\pi$

Examination of the $\alpha = \frac{1}{4}\pi$ waveform in figure 12.2b shows the voltage waveform is made from five sinusoidal segments. The rms load voltage per phase (line to neutral) is

$$V_{rms} = \hat{V}\left[\frac{1}{\pi}\left\{\begin{array}{l}\int_{\alpha}^{\pi/3}\frac{1}{3}\sin^2\phi\;d\phi + \int_{\frac{1}{3}\pi}^{\frac{1}{3}\pi+\alpha}\frac{1}{4}\sin^2(\phi - \frac{1}{6}\pi)\;d\phi + \int_{\frac{1}{3}\pi+\alpha}^{\frac{2}{3}\pi}\frac{1}{3}\sin^2\phi\;d\phi \\ \quad + \int_{\frac{2}{3}\pi}^{\frac{2}{3}\pi+\alpha}\frac{1}{4}\sin^2(\phi - \frac{1}{6}\pi)\;d\phi + \int_{\frac{2}{3}\pi+\alpha}^{\pi}\frac{1}{3}\sin^2\phi\;d\phi\end{array}\right\}\right]^{\frac{1}{2}}$$

$$V_{rms} = I_{rms}R = V\left[1 - \tfrac{3}{2\pi}\alpha + \tfrac{3}{4\pi}\sin 2\alpha\right]^{\frac{1}{2}}$$

For phase delay angles $\frac{1}{3}\pi \le \alpha \le \frac{1}{2}\pi$

Examination of the $\alpha = \frac{1}{3}\pi$ or $\alpha = \frac{1}{2}\pi$ waveforms in figure 12.2b show the voltage waveform is comprised from two segments. The rms load voltage per phase is

$$V_{rms} = \hat{V}\left[\frac{1}{\pi}\left\{\int_{\alpha}^{\frac{1}{3}\pi+\alpha}\frac{1}{4}\sin^2(\phi - \frac{1}{6}\pi)\;d\phi + \int_{\frac{1}{3}\pi+\alpha}^{\frac{1}{3}\pi+\alpha}\frac{1}{4}\sin^2(\phi - \frac{1}{6}\pi)\;d\phi\right\}\right]^{\frac{1}{2}}$$

$$V_{rms} = I_{rms}R = V\left[\tfrac{1}{2} + \tfrac{9}{8\pi}\sin 2\alpha + \tfrac{3\sqrt{3}}{8\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

For phase delay angles $\frac{1}{2}\pi \le \alpha \le \frac{5}{6}\pi$

Examination of the $\alpha = \frac{3}{4}\pi$ waveform in figure 12.2b shows the voltage waveform is made from two segments. The rms load voltage per phase is

$$V_{rms} = \hat{V}\left[\frac{1}{\pi}\left\{\int_{\alpha}^{\frac{5}{6}\pi}\tfrac{1}{4}\sin^2(\phi - \tfrac{1}{6}\pi)\ d\theta + \int_{\frac{1}{3}\pi+\alpha}^{\frac{7}{6}\pi}\tfrac{1}{4}\sin^2(\phi - \tfrac{1}{6}\pi)\,d\phi\right\}\right]^{\frac{1}{2}}$$

$$V_{rms} = I_{rms}R = V\left[\tfrac{5}{4} - \tfrac{3}{2\pi}\alpha + \tfrac{3}{8\pi}\sin 2\alpha + \tfrac{3\sqrt{3}}{8\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

In each case the phase current and line to line voltage are related by $V_{Lrms} = \sqrt{3}I_{rms}R$ and $\hat{V} = \sqrt{2}\,V_L = \sqrt{6}\,V$.

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12.2.2 Half-controlled three-phase ac regulator

The half-controlled three-phase regulator shown in figure 12.3a requires only a single trigger pulse per thyristor and the return path is via a diode. Compared with the fully controlled regulator, the half-controlled regulator is simpler and does not give rise to dc components but does produce more line harmonics.

Figure 12.3b shows resistive symmetrical load, line-to-neutral voltage waveforms for four different phase delay angles, α. Three distinctive conduction periods exist.

i. $0 \le \alpha \le \tfrac{1}{3}\pi$

Before turn-on, one diode and one thyristor conduct in the other two phases. After turn-on two thyristors and one diode conduct, and the three-phase ac supply is impressed across the load. Examination of the $\alpha = \tfrac{1}{4}\pi$ waveform in figure 12.3b shows the voltage waveform is made from three segments. The rms load voltage per phase (line to neutral) is

$$V_{rms} = I_{rms}R = V\left[1 + \tfrac{3\alpha}{4\pi} - \tfrac{3}{8\pi}\sin 2\alpha\right]^{\frac{1}{2}} \qquad (12.15)$$
$$0 \le \alpha \le \tfrac{1}{2}\pi$$

ii. $\tfrac{1}{3}\pi \le \alpha \le \tfrac{2}{3}\pi$

Only one thyristor conducts at one instant and the return current is shared at different intervals by one ($\tfrac{1}{3}\pi \le \alpha \le \tfrac{1}{2}\pi$) or two ($\tfrac{1}{2}\pi \le \alpha \le \tfrac{2}{3}\pi$) diodes. Examination of the $\alpha = \tfrac{3}{8}\pi$ and $\alpha = \tfrac{5}{8}\pi$ waveforms in figure 12.3b show the voltage waveform is made from three segments, although different segments of the supply around $\omega t=\pi$. The rms load voltage per phase (line to neutral) in the first conducting case is given by equation (12.15) while after $\alpha=\tfrac{1}{2}\pi$ the rms voltage is

$$V_{rms} = I_{rms}R = V\left[\left\{\tfrac{11}{8} - \tfrac{3\alpha}{2\pi}\right\}\right]^{\frac{1}{2}} \qquad (12.16)$$
$$\tfrac{1}{2}\pi \le \alpha \le \tfrac{2}{3}\pi$$

iii. $\tfrac{2}{3}\pi \le \alpha \le 7\pi/6$

Current flows in only one thyristor and one diode and at $7\pi/6$ zero power is delivered to the load. The output voltage waveform shown for $\alpha=\tfrac{3}{4}\pi$ in figure 12.3b has one component.

$$V_{rms} = I_{rms}R = V\left[\tfrac{7}{8} - \tfrac{3}{4\pi}\alpha + \tfrac{3}{16}\sin 2\alpha - \tfrac{3\sqrt{3}}{16}\cos 2\alpha\right]^{\frac{1}{2}} \qquad (12.17)$$
$$\tfrac{2}{3}\pi \le \alpha \le \tfrac{7}{6}\pi$$

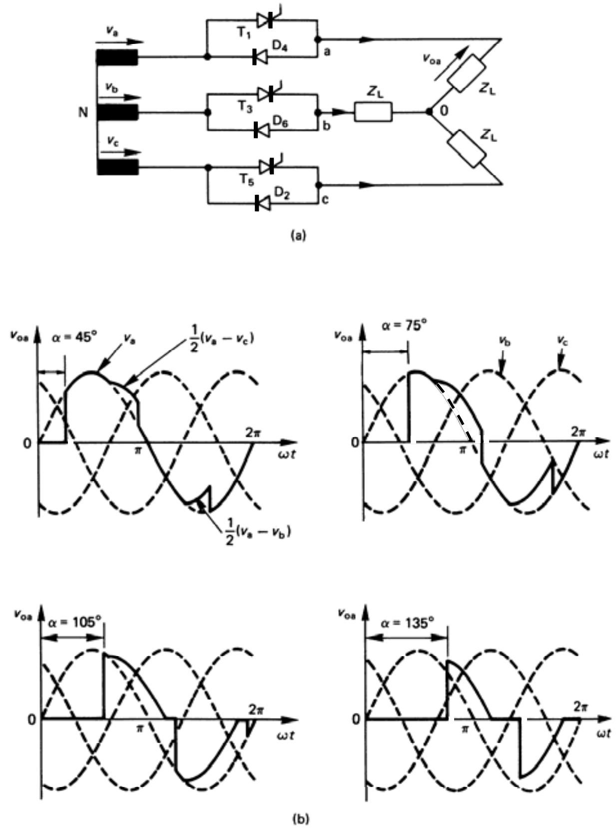

Figure 12.3. *Three-phase half-wave ac voltage regulator: (a) circuit connection with a star load and (b) phase a, line-to-load neutral voltage waveforms for four firing delay angles.*

For delta-connected loads where each phase end is accessible, the regulator shown in figure 12.4 can be employed in order to reduce thyristor current ratings. The phase rms voltage is given by

$$V_{rms} / phase = \sqrt{2}V\left[1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}\right] \qquad 0 \le \alpha \le \pi \tag{12.18}$$

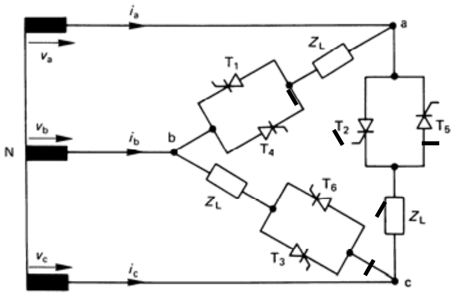

Figure 12.4. *A delta connected three-phase ac regulator.*

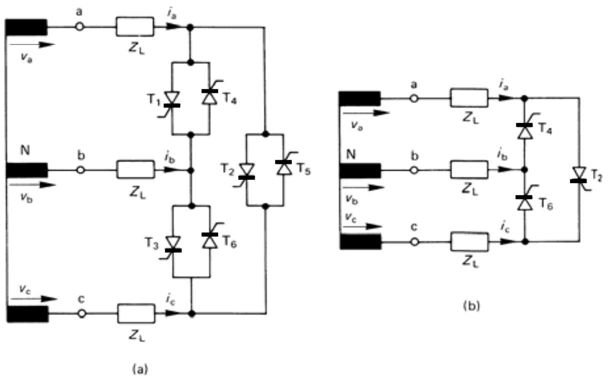

Figure 12.5. *Open-star three-phase ac regulators:*
(a) with six thyristors and (b) with three thyristors.

For star-connected loads where access exists to a neutral that can be opened, the regulator in figure l2.5a can be used. This circuit produces identical load waveforms to those for the regulator in figure 12.2, except that the device current ratings are halved. Only one thyristor needs to be conducting for load current, compared with the circuit of figure 12.2 where two devices must be triggered.

The number of devices and control requirements for the regulator of figure 12.5a can be simplified by employing the regulator in figure 12.5b. Another simplification, at the expense of harmonics, is to connect one phase of the load in figure l2.2a directly to the supply, thereby eliminating a pair of line thyristors.

12.3 Integral cycle control

In thyristor heating applications, load harmonics are unimportant and integral cycle control, or burst firing, can be employed. Figure l2.6a shows the regulator when a triac is employed and figure 12.6b shows the output voltage indicating the regulator's operating principle.

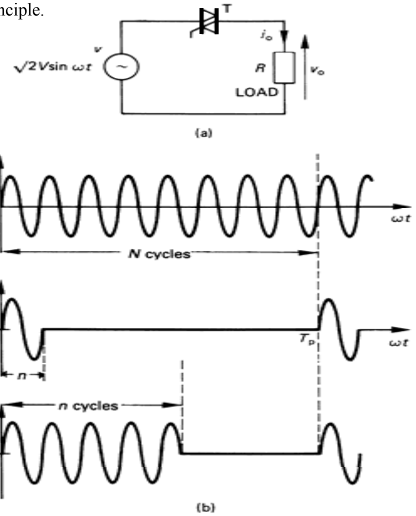

Figure 12.6. *Integral half-cycle single-phase ac control:*
(a) circuit connection using a triac; (b) output voltage waveforms for one-eighth maximum load power and nine-sixteenths maximum power.

In many heating applications the load thermal time constant is long (relative to 20ms, that is 50Hz) and an acceptable control method involves a number of mains cycles on and then off. Because turn-on occurs at zero voltage cross-over and turn-off occurs at zero current, which is near a zero voltage cross-over, supply harmonics and radio frequency interference are low. The lowest order harmonic in the load is $1/T_p$.
The rms output voltage is

$$V_{rms} = \left(\frac{1}{2\pi} \int_0^{2\pi n/N} \left(\sqrt{2} V \sin N\omega t \right)^2 d\omega t \right) \tag{12.19}$$

$$V_{rms} = V\sqrt{n/N}$$

The output power is

$$P = \frac{V^2}{R}\frac{n}{N} \quad \text{(W)} \tag{12.20}$$

where n is the number of on cycles and N is the number of cycles in the period T_p. Finer resolution output voltage control is achievable if integral half-cycles are used rather than full cycles. The average and rms thyristors currents are, respectively,

$$\bar{I}_{Th} = \frac{\sqrt{2}V}{\pi R}\frac{n}{N} \qquad I_{Th_{rms}} = \frac{\sqrt{2}V}{2R}\sqrt{\frac{n}{N}} \tag{12.21}$$

From these two equations the distortion factor μ is $\sqrt{n/N}$ and the power transfer ratio is n/N. The supply displacement factor $\cos\psi$ is unity and supply power factor λ is $\sqrt{n/N}$, shown in figure 12.6b. The rms voltage at the supply frequency is $V\,n/N$. The equations remain valid if integral half cycle control is used. The introduction of sub-harmonics tends to restrict this control technique to resistive heating type application. Temperature effects on load resistance R have been neglected.

Example 12.3: *Integral cycle control*

The power delivered to a 12Ω resistive heating element is derived from an ideal sinusoidal supply √2 240 sin 2π 50 t and is controlled by a series connected triac as shown in figure 12.6. The triac is controlled from its gate so as to deliver integral ac cycle pulses of three (n) consecutive ac cycles from four (N).
Calculate

i. The percentage power transferred compared to continuous ac operation
ii. The supply power factor, distortion factor, and displacement factor
iii. The supply frequency (50Hz) harmonic component voltage of the load voltage
iv. The triac maximum *di/dt* and *dv/dt* stresses
v. The phase angle α, to give the same load power when using phase angle control. Compare the maximum *di/dt* and *dv/dt* stresses with part iv.
vi. The output power steps when n, the number of conducted cycles is varied with respect to N = 4 cycles. Calculate the necessary phase control α equivalent for the same power output. Include the average and rms thyristor currents.
vii. What is the smallest power increment if half cycle control were to be used?

Solution
The key data is

$n = 3$
$N = 4$
V = 240 rms ac, 50Hz

i. The power transfer, given by equation (12.20), is

$$P = \frac{V^2}{R}\frac{n}{N} = \frac{240^2}{12\Omega} \times ¾ = 4800 \times ¾ = 3.6\text{kW}$$

That is 75% of the maximum power is transferred to the load as heating losses.

ii. The displacement factor, $\cos\psi$, is 1. The distortion factor is given by

$$\mu = \sqrt{\frac{n}{N}} = \sqrt{\frac{3}{4}} = 0.866$$

Thus the supply power factor, λ, is

$$\lambda = \mu\cos\psi = \sqrt{\frac{n}{N}} = 0.866 \times 1 = 0.866$$

iii. The 50Hz rms component of the load voltage is given by

$$V_{50Hz} = V\frac{n}{N} = 240 \times ¾ = 180\text{V rms}$$

iv. The maximum *di/dt* and *dv/dt* occur at zero cross over, when $t = 0$.

$$\left.\frac{dV_s}{dt}\right|_{\max} = \frac{d}{dt}\sqrt{2}\;240\sin 2\pi 50t\,\Big|_{t=0}$$
$$= \sqrt{2}\;240\,(2\pi 50)\cos 2\pi 50t\big|_{t=0}$$
$$= \sqrt{2}\;240\,(2\pi 50) = 0.107\ \text{V/μs}$$
$$\left.\frac{d}{dt}\frac{V_s}{R}\right|_{\max} = \frac{d}{dt}\frac{\sqrt{2}\;240}{12\Omega}\sin 2\pi 50t\,\Big|_{t=0}$$
$$= \sqrt{2}\;20\,(2\pi 50)\cos 2\pi 50t\big|_{t=0}$$
$$= \sqrt{2}\;20\,(2\pi 50) = 8.89\ \text{A/ms}$$

v. To develop the same load power, 3600W, with phase angle control, with a purely resistive load, implies that both methods must develop the same rms current and voltage, that is, $V_{rms} = \sqrt{R\,P} = V\sqrt{n/N}$. From equation (12.4), when the extinction angle, $\beta = \pi$, since the load is resistive

$$V_{rms} = \sqrt{R \times P} = V\sqrt{n/N} = V\left[\tfrac{1}{\pi}\left\{ (\pi - \alpha) + ½\sin 2\alpha \right\} \right]^{½}$$

that is

$$\frac{n}{N} = \tfrac{1}{\pi}\left\{(\pi-\alpha)+\tfrac{1}{2}\sin 2\alpha\right\}$$
$$= \tfrac{3}{4} = \tfrac{1}{\pi}\left\{(\pi-\alpha)+\tfrac{1}{2}\sin 2\alpha\right\}$$

Solving $0 = \tfrac{1}{4}\pi - \alpha + \tfrac{1}{2}\sin 2\alpha$ iteratively gives $\alpha = 63.9°$.
When the triac turns on at α = 63.9°, the voltage across it drops virtually instantaneously from √2 240 sin 63.9 = 305V to zero. Since this is at triac turn-on, this very high *dv/dt* does not represent a turn-on *dv/dt* stress. The maximum triac *dv/dt* stress tending to turn it on is at zero voltage cross over, which is 107 V/ms, as with integral cycle control. Maximum *di/dt* occurs at triac turn on where the current rises from zero amperes to 305V/12Ω = 25.4A quickly. If the triac turns on in approximately 1µs, then this would represent a *di/dt* of 25.4A/µs. The triac initial *di/dt* rating would have to be in excess of 25.4A/µs.

cycles	period	power	$\bar{I}_{Th}$	$I_{Th_{rms}}$	Delay angle	Displacement factor	Distortion factor	Power factor
n	N	W	A	A	α	cosψ	μ	λ
0	4	0	0	0	180°			
1	4	1200	2.25	7.07	114°	1	½	½
2	4	2400	4.50	10.0	90°	1	0.707	0.707
3	4	3600	6.75	12.2	63.9°	1	0.866	0.866
4	4	4800	9	14.1	1	1	1	1

vi. The output power can be varied using n = 0, 1, 2, 3, or 4 cycles of the mains. The output power in each case is calculated as in part 1 and the equivalent phase control angle, α, is calculated as in part v. The appropriate results are summarised in the table.

vii. Finer power step resolution can be attained if half cycle power pulses are used as in figure 12.6b. If one complete ac cycle corresponds to 1200W then by using half cycles, 600W power steps are possible. This results in nine different power levels if N = 4.

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12.4 Single-phase transformer tap-changer

Figure 12.7 shows a single-phase tap changer where the tapped ac voltage supply can be provided by a tapped transformer or autotransformer.
Thyristor T_3 (T_4) is triggered at zero voltage cross-over, then under phase control T_1 (T_2) is turned on. The output voltage for a resistive load is defined by

$$v_o = \sqrt{2}\,V_2 \sin\omega t \quad \text{(V)} \qquad \text{for } 0 \le \omega t \le \alpha \quad \text{(rad)} \tag{12.22}$$

$$v_o = \sqrt{2}\,V_1 \sin\omega t \quad \text{(V)} \qquad \text{for } \alpha \le \omega t \le \pi \quad \text{(rad)} \tag{12.23}$$

where α is the phase delay angle and $v_2 < v_1$.
For a resistive load the rms output voltage is

$$V_{rms} = \left[\frac{V_2^2}{\pi}\left(\alpha - \tfrac{1}{2}\sin 2\alpha\right) + \frac{V_1^2}{\pi}\left(\pi - \alpha + \tfrac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} \tag{12.24}$$

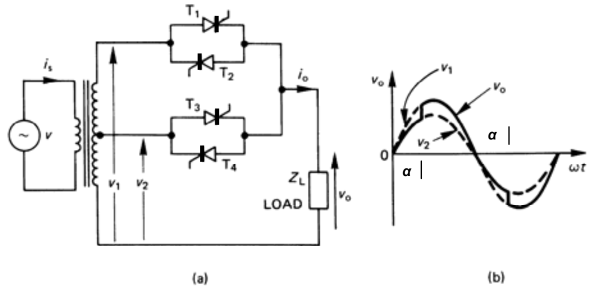

Figure 12.7. *An ac voltage regulator using a tapped transformer: (a) circuit connection and (b) output voltage waveform with a resistive load.*

Initially v_2 is impressed across the load. Turning on T_1 (T_2) reverse-biases T_3 (T_4), hence T_3 (T_4) turns off and the load voltage jumps to v_1. It is possible to vary the rms load voltage between v_2 and v_1. It is important that T_1 (T_2) and T_4 (T_3) do not conduct simultaneously, since such conduction short-circuits the transformer secondary.
With an inductive load circuit, when only T_1 and T_2 conduct, the output current is

$$i_o = \frac{\sqrt{2}\,V}{Z}\sin(\omega t - \phi) \quad \text{(A)} \tag{12.25}$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ (ohms) $\quad \phi = \tan^{-1} \omega L / R$ (rad)
It is important that T_3 and T_4 are not fired until $\alpha \ge \phi$, when the load current must have reached zero. Otherwise a transformer secondary short circuit occurs through T_1 (T_2) and T_4 (T_3).
For a resistive load, the thyristor rms currents for T_3, T_4 and T_1, T_2 respectively are

$$I_{Trms} = \frac{v_2}{2R}\sqrt{\frac{1}{\pi}\left(2\alpha - \sin 2\alpha\right)}$$
$$I_{Trms} = \frac{v_1}{2R}\sqrt{\frac{1}{\pi}\left(\sin 2\alpha - 2\alpha\right) + 2\pi} \qquad (12.26)$$

The thyristor voltages ratings are both v_1 - v_2, provided a thyristor is always conducting at any instant.

An extension of the basic operating principle is to use phase control on thyristors T_3 and T_4 as well as T_1 and T_2. It is also possible to use tap-changing in the primary circuit. The basic principle can also be extended from a single tap to a multi-tap transformer.

The basic operating principle of any multi-output tap changer, in order to avoid short circuits, independent of the load power factor is

- switch up in voltage when the load *V* and *I* have the same direction, delivering power
- switch down when *V* and *I* have the opposite direction, returning power.

12.5 Cycloconverter

The simplest cycloconverter is a single-phase ac input to single-phase ac output circuit as shown in figure 12.8a. It synthesises a low-frequency ac output from selected portions of a higher-frequency ac voltage source and consists of two converters connected back-to-back. Thyristors T_1 and T_2 form the positive converter group P, while T_3 and T_4 form the negative converter group N.

Figure 12.8b shows how an output frequency of one-fifth of the input supply frequency is generated. The P group conducts for five half-cycles (with T_1 and T_2 alternately conducting), then the N group conducts for five half-cycles (with T_3 and T_4 alternately conducting). The result is an output voltage waveform with a fundamental of one-fifth the supply with continuous load and supply current.

The harmonics in the load waveform can be reduced and rms voltage controlled by using phase control as shown in figure 12.8c. The phase control delay angle is greater towards the group changeover portions of the output waveform. The supply current is now distorted and contains a subharmonic at the cycloconverter output frequency, which for figure 12.8c is at one-fifth the supply frequency.

With inductive loads, one blocking group cannot be turned on until the load current through the other group has fallen to zero, otherwise the supply will be short-circuited. An intergroup reactor, L, as shown in figure 12.8a can be used to limit any intergroup circulating current, and to maintain a continuous load current.

A single-phase ac load fed from a three-phase ac supply, and three-phase ac load cycloconverters can also be realised as shown in figures 12.9a and 12.9b, respectively.

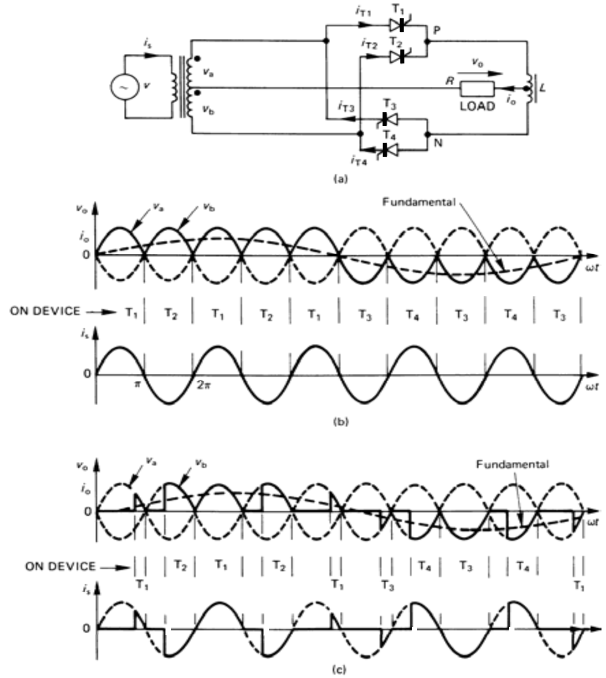

Figure 12.8. *Single-phase cycloconverter ac regulator: (a) circuit connection with a purely resistive load; (b) load voltage and supply current with 180° conduction of each thyristor; and (c) waveforms when phase control is used on each thyristor.*

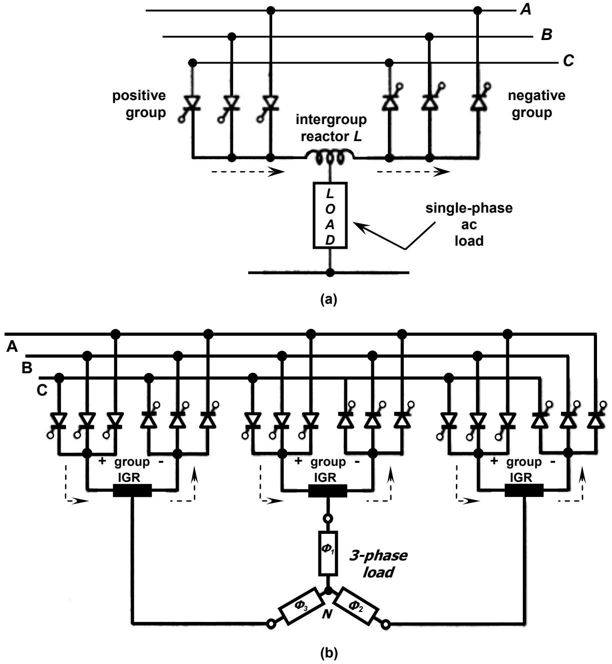

Figure 12.8. *Cycloconverter ac regulator circuits: (a) three-phase to single-phase and (b) three–phase supply to three-phase load.*

12.6 The matrix converter

Commutation of the cycloconverter switches is restricted to natural commutation instances dictated by the supply voltages. This usually results in the output frequency being significantly less than the supply frequency if reasonable low harmonic output is required. In the matrix converter in figure 12.9b, the thyristors in figure 12.8b are replaced with fully controlled, bidirectional switches, like that shown in figure 12.9a.

Rather than eighteen switches and eighteen diodes, nine switches and thirty-six diodes can be used if a unidirectional voltage and current switch in a full-bridge configuration is used as shown in figure 6.11. These switch configurations allow converter current commutation as and when desired, provide certain condition are fulfilled. These switches allow any one input supply ac voltage and current to be directed to any one or more output lines. At any instant only one of the three input voltages can be connected to a given output. This flexibility implies a higher quality output voltage can be attained, with enough degrees of freedom to ensure the input currents are sinusoidal and with unity (or adjustable) power factor. The input *L-C* filter prevents matrix modulation frequency components from being injected into the input three-phase ac supply system.

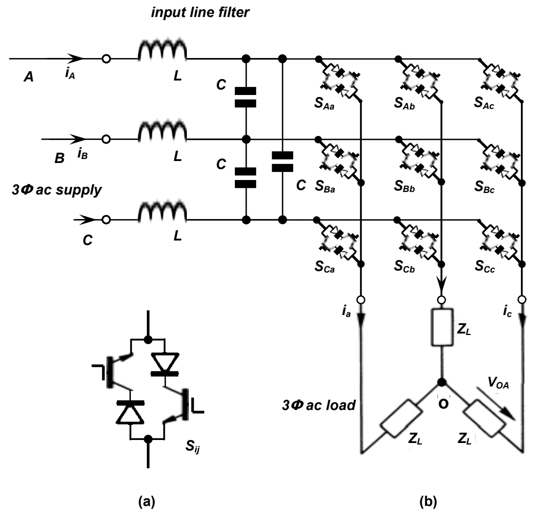

Figure 12.9. Three-phase input to three-phase output matrix *converter circuit: (a) bidirectional switch and (b) three–phase ac supply to three-phase ac load.*

The relationship between the output voltages (v_a, v_b, v_c) and the input voltages (v_A, v_B, v_C) is determined by the states of the nine bidirectional switches ($S_{i,j}$), according to

$$\begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix} = \begin{pmatrix} S_{Aa} & S_{Ba} & S_{Ca} \\ S_{Ab} & S_{Bb} & S_{Cb} \\ S_{Ac} & S_{Bc} & S_{Cc} \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} \quad \text{(A)} \qquad (12.27)$$

With the balanced star load shown in figure 12.1c, the load neutral voltage v_o is given by

$$v_o = \tfrac{1}{3}\left(v_a + v_b + v_c\right) \qquad (12.28)$$

The line-to-neutral and line-to-line voltages are the same as those applicable to svm, namely

$$\begin{pmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix} \quad \text{(V)} \qquad (12.29)$$

from which

$$\begin{pmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{pmatrix} = \tfrac{1}{2} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix} \quad \text{(V)} \qquad (12.30)$$

Similarly the relationship between the input line currents (i_A, i_B, i_C) and the output currents (i_a, i_b, i_c) is determined by the states of the nine bidirectional switches ($S_{i,j}$), according to

$$\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} S_{Aa} & S_{Ab} & S_{Ac} \\ S_{Ba} & S_{Bb} & S_{Bc} \\ S_{Ca} & S_{Cb} & S_{Cc} \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} \quad \text{(A)} \qquad (12.31)$$

where the switches S_{ij} are constrained such that no two or three switches short the input lines or would cause discontinuous output current. Discontinuous output current cannot occur since no natural current freewheel paths exist. The input short circuit constraint is complied with by ensuring that only one switch in each column of the matrix in equation (12.31) is on at any time. Thus not all the 512 (2^9) states can be used, and only 27 states of the switch matrix can be utilised.
The maximum voltage gain, the ratio of the peak fundamental ac output voltage to the peak ac input voltage is ½√3 = 0.866. Above this level, called over-modulation, distortion of the input current occurs. Since the switches are bidirectional and fully controlled, power flow can be bidirectional. Control involves the use of a modulation index that varies sinusoidally. Since no intermediate energy storage stage is involved, such as a dc link, this total silicon solution to ac to ac conversion provides no ride-through, thus is not well suited to ups application.

12.7 Load efficiency and supply current power factor

One characteristic of ac regulators is non-sinusoidal load, hence supply current as illustrated in figure 12.lb. Difficulty therefore exists in defining the supply current power factor and the harmonics in the load current may detract from the load efficiency. For example, with a single-phase motor, current components other than the fundamental detract from the fundamental torque and increase motor heating, noise, and vibration. To illustrate the procedure for determining load efficiency and supply power factor, consider the circuit and waveforms in figure 12.1.

12.7.1 Load waveforms

The load voltage waveform is constituted from the sinusoidal supply voltage v and is defined by

$$\begin{aligned} v_o(\omega t) &= \sqrt{2}V \sin \omega t \quad \text{(V)} \\ &\alpha \le \omega t \le \beta \\ &\pi + \alpha \le \omega t \le \pi + \beta \end{aligned} \qquad (12.32)$$

and $v_o = 0$ elsewhere.
Fourier analysis of v_o yields the load voltage Fourier coefficients v_{an} and v_{bn} such that

$$v_o(\omega t) = \sum \{ v_{an} \cos n\omega t + v_{bn} \sin n\omega t \} \quad \text{(V)} \qquad (12.33)$$

for all values of n.
The load current can be evaluated by solving

$$Ri_o + L\frac{di_o}{dt} = \sqrt{2}\, V \sin \omega t \quad \text{(V)} \qquad (12.34)$$

over the appropriate bounds and initial conditions. From Fourier analysis of the load current i_o, the load current coefficients i_{an} and i_{bn} can be derived.
Derivation of the current waveform Fourier coefficients may prove complicated because of the difficulty of integrating an expression involving equation (12.2), the load current. An alternative and possibly simpler approach is to use the fact that each load Fourier voltage component produces a load current component at the associated frequency but displaced because of the load impedance at that frequency. That is

$$\begin{aligned} i_{an} &= \tfrac{v_{an}}{R}\cos\phi_n \quad \text{(A)} \\ i_{bn} &= \tfrac{v_{bn}}{R}\cos\phi_n \quad \text{(A)} \end{aligned} \qquad (12.35)$$

$$\text{where} \quad \phi_n = \tan^{-1} {n\omega L}/{R}$$

The load current i_o is given by

$$i_o(\omega t) = \sum_{\forall n} \left\{ i_{an} \cos\left(n\omega t - \phi_n\right) + i_{bn} \sin\left(n\omega t - \phi_n\right) \right\} \quad \text{(A)} \qquad (12.36)$$

The load efficiency, η, which is related to the power dissipated in the resistive component R of the load, is defined by

$$\eta = \text{fundamental active power} \big/ \text{total active power}$$

$$= \frac{\frac{1}{2}\left(i_{a1}^2 R + i_{b1}^2 R\right)}{\frac{1}{2}\sum_{\forall n}\left(i_{an}^2 R + i_{bn}^2 R\right)} = \frac{i_{a1}^2 + i_{b1}^2}{\sum_{\forall n}\left(i_{an}^2 + i_{bn}^2\right)} \qquad (12.37)$$

In general, the total load power is $\sum_{\forall n} v_{n\,rms} \times i_{n\,rms} \times \cos\phi_n$.

12.7.2 Supply waveforms

i. The supply distortion factor μ, displacement factor $\cos\psi$, and power factor λ give an indication of the adverse effects that a non-sinusoidal load current has on the supply as a result of thyristor phase control.
In the circuit of figure 12.1a, the load and supply currents are the same and are given by equation (12.2). The supply current Fourier coefficients i_{san} and i_{sbn} are the same as for the load current Fourier coefficients i_{sa} and i_{sb} respectively, as previously defined.
The total supply power factor λ can be defined as

$$\lambda = \text{real power} \big/ \text{apparent power} = \text{total mean input power} \big/ \text{total rms input VA}$$

$$= \frac{v_{1rms} i_{1rms} \cos\psi_1}{V_{rms} I_{rms}} = \frac{\frac{1}{\sqrt{2}}\sqrt{v_{sa1}^2 + v_{sb1}^2} \times \frac{1}{\sqrt{2}}\sqrt{i_{sa1}^2 + i_{sb1}^2} \times \cos\psi_1}{v \times \frac{1}{\sqrt{2}}\sqrt{i_{a1}^2 + i_{b1}^2}} \qquad (12.38)$$

The supply voltage is sinusoidal hence supply power is not associated with the harmonic non-fundamental currents.

$$\lambda = \frac{v\sqrt{\frac{1}{2}\left(i_{sa1}^2 + i_{sb1}^2\right)}\ \cos\psi_1}{v\, i_{rms}}$$

$$= \frac{\sqrt{\frac{1}{2}\left(i_{sa1}^2 + i_{sb1}^2\right)}\ \cos\psi_1}{i_{rms}} \qquad (12.39)$$

where cos ψ, termed the displacement factor, is the fundamental power factor defined as

$$\cos\psi_1 = \cos\left(-\tan^{-1} i_{sa1} \big/ i_{sb1}\right) \qquad (12.40)$$

Equating with equation (12.39), the total supply power factor is defined as

$$\lambda = \mu \cos\psi_1 \qquad (12.41)$$

The supply current distortion factor μ is the ratio of fundamental rms current to total rms current i_{srms}, that is

$$\mu = \frac{\sqrt{\frac{1}{2}\left(i_{sa1}^2 + i_{sb1}^2\right)}}{i_{rms}} \qquad (12.42)$$

ii. The supply harmonic factor ρ is defined as

$$\rho = \frac{\text{total harmonic rms current (or voltage)}}{\text{fundamental rms current (or voltage)}}$$

$$= I_h \big/ I_{1rms} = I_h \big/ \sqrt{i_{sa1}^2 + i_{sb1}^2} = \sqrt{\frac{i_{s_{rms}}^2}{i_{sa1}^2 + i_{sb1}^2} - 1} \qquad (12.43)$$

where I_h is the total harmonic current,

$$I_h = \sqrt{I_{rms}^2 - I_{1rms}^2}$$

$$= \sqrt{\sum_{n=2}^{\infty} I_{nrms}^2} = \frac{1}{\sqrt{2}}\sqrt{\sum_{n=2}^{\infty} i_{san}^2 + i_{sbn}^2} \qquad (12.44)$$

iii. The supply crest factor δ is defined as the ratio of peak supply current $\hat{i}_s$ to the total rms current, that is

$$\delta = \hat{i}_s / I_{rms} \qquad (12.45)$$

iv. The energy conversion factor υ is defined by

$$\upsilon = \frac{\text{fundamental output power}}{\text{fundamental input power}}$$

$$= \frac{\frac{1}{\sqrt{2}}\sqrt{v_{a1}^2 + v_{b1}^2} \times \frac{1}{\sqrt{2}}\sqrt{i_{a1}^2 + i_{b1}^2} \times \cos\phi_1}{v \times \frac{1}{\sqrt{2}}\sqrt{i_{sa1}^2 + i_{sb1}^2} \times cos\psi_1} \qquad (12.46)$$

Example 12.4: ***Load efficiency***

If a purely resistive load R is fed with a voltage

$$v_o = \sqrt{2}\, V \sin\omega t + \frac{\sqrt{2}\, V}{3} \sin 3\omega t$$

what is the fundamental load efficiency?

Solution
The load current is given by

$$i_o = v_o \big/ R = \frac{\sqrt{2}\, V}{R}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t\right)$$

The load efficiency is given by equation (12.37), that is

$$\eta = \frac{\left(\frac{\sqrt{2}V}{R}\right)^2 R}{\left(\frac{\sqrt{2}V}{R}\right)^2 R + \left(\frac{\sqrt{2}V}{3R}\right)^2 R}$$

$$= \frac{1}{1+ {}^{1}\!/_{9}} = 0.9$$

The introduced third harmonic component decreases the load efficiency by 10%.

♣

Reading list

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Dewan, S. B. and Straughen, A., *Power Semiconductor Circuits*, John Wiley and Sons, New York, 1975.

General Electric Company, *SCR Manual*, 6th Edition, 1979.

Shepherd, W., *Thyristor Control of AC Circuits*, Granada, 1975.

Problems

12.1. Determine the rms load current for the ac regulator in figure 12.3, with a resistive load *R*. Consider the delay angle intervals 0 to ½π, ½π to ⅔π and ⅔π to 7π /6.

12.2. The ac regulator in figure 12.3, with a resistive load *R* has one thyristor replaced by a diode. Show that the rms output voltage is

$$V_{rms} = \left[\frac{1}{2\pi}\left(2\pi - \alpha + \tfrac{1}{2}\sin 2\alpha\right)\right]^{1/2}$$

while the average output voltage is

$$\overline{V}_o = \frac{\sqrt{2}\,V}{2\pi}\left(\cos\alpha - 1\right)$$

12.3. Plot the load power for a resistive load for the fully controlled and half-controlled three-phase ac regulator, for varying phase delay angle, α. Normalise power with respect to $\hat{V}^2 / R$.

12.4. For the tap changer in figure 12.7, with a resistive load, calculate the rms output voltage for a phase delay angle α. If v_2 = 200 V ac and v_1 = 240 V ac, calculate the power delivered to a 10 ohm resistive load at delay angles of ¼π, ½π, and ¾π. What is the maximum power that can be delivered to the load?

12.5. A. 0.01 H inductance is added in series with the load in problem 12.4. Determine the load voltage and current waveforms at a firing delay angle of ½π. Assuming a 50 Hz supply, what is the minimum delay angle?

12.6. The thyristor T_2 in the single phase controller in figure 12.la is replaced by a diode. The supply is 240 V ac, 50 Hz and the load is 10 Ω resistive. For a delay angle of α = 90°, determine the

i. rms output voltage
ii. supply power factor
iii. mean output voltage
iv. mean input current.

[207.84 V; 0.866 lagging; 54 V; 5.4 A]

12.7. The single phase ac controller in figure 12.6 operating on the 240 V, 50 Hz mains is used to control a 10 Ω resistive heating load. If the load is supplied repeatedly for 75 cycles and disconnected for 25 cycles, determine the

i. rms load voltage,
ii. input power factor, λ, and
iii. the rms thyristor current.

12.8. The ac controller in problem 12.7 delivers 2.88 kW. Determine the duty cycle, *n/N*, and the input power factor, λ.