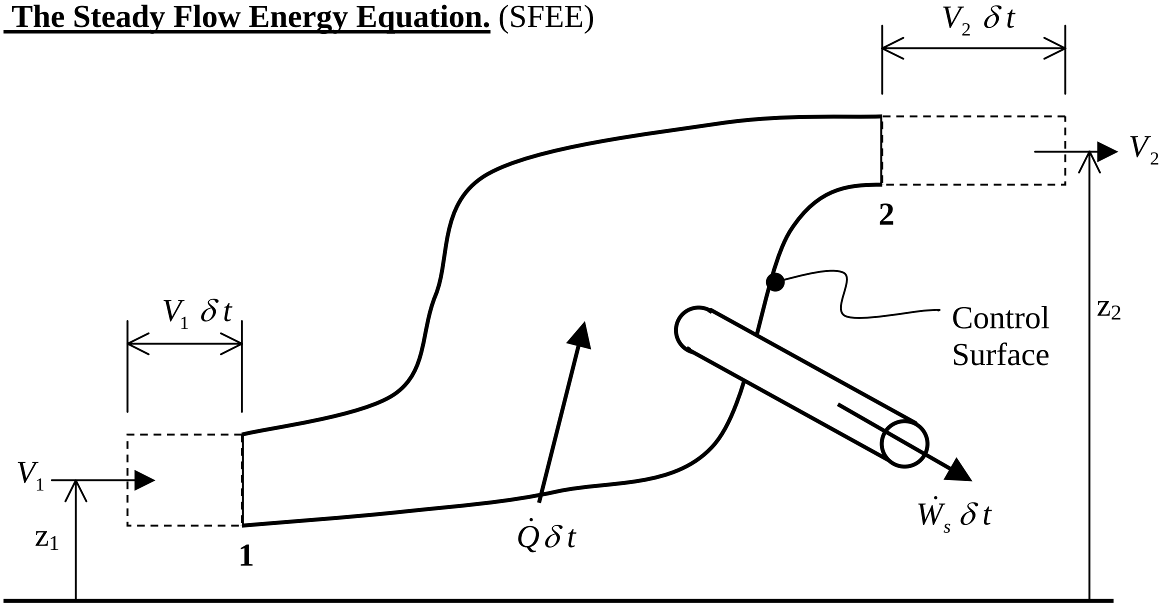
Chapter 6:

Energy Conservation in Steady Flow.

The Steady Flow Energy Equation. (SFEE)



The sketch above shows a piece of equipment such as a boiler, engine, pump, etc. through which fluid steadily flows. The fluid enters the equipment with velocity V_1 at an inlet **1** with area A_1 and leaves with velocity V_2 by an exhaust at **2** with area A_2. The heights of the inlet and exhaust above some reference datum are z_1 and z_2 respectively.

Note:

I have a problem with symbols here. I would like to use a different font for velocity and volume so that they can be distinguished from each other but that is not within the capacity of the equation setter. For this part I shall avoid the use of total volume so that a capital V is velocity. Usually it is clear from the context whether the V represents volume or velocity.

The equipment is enclosed within an imaginary surface called a control surface. Note that, unlike a system boundary, matter can cross a control surface. The control surface encloses a *control volume* and the objective here is to undertake an energy balance for that control volume.

Heat is transferred across the control surface at rate $\dot{Q}$ and work is being performed at rate $\dot{W}_s$ by an output shaft. That work is called shaft work. As will be seen $\dot{W}_s \neq \dot{W}$.

Consider a system that initially fills the control volume and a small section of the inlet plumbing outside the control volume. A small time δt later that system has moved so that it now coincides precisely with the control surface at the inlet, but part of the system has left the control volume at the exhaust.

The mass entering the control volume in time δt is $\delta m = \rho_1 V_1 A_1 \delta t$
The mass leaving the control volume in time δt is $\delta m = \rho_2 V_2 A_2 \delta t$

Since the flow is steady mass is not accumulating inside the control volume so

$$\boxed{\rho_1 V_1 A_1 = \rho_2 V_2 A_2}$$

This is called the *continuity equation*.

Let the stored energy inside the control volume be *E*. Then the initial stored energy in the system will be:

$$E_1 = g z_1 \delta m + \tfrac{1}{2}\delta m V_1^2 + \delta m u_1 + E$$

Similarly the final stored energy would be

$$E_2 = g z_2 \delta m + \tfrac{1}{2}\delta m V_2^2 + \delta m u_2 + E$$

So for the system $\Delta E = \delta m \Delta\left(u + \tfrac{1}{2}V^2 + g z\right)$

Note here that the external energy is being taken into account explicitly.

At the inlet the fluid outside the system pushes the system into the control volume. The work done on the system is called *flow work* and is given by $-p_1 A_1 V_1 \delta t = -p_1 v_1 \delta m$
The negative sign is because the sign convention makes work done on the system negative.

Similarly at the exhaust the flow work is $p_2 A_2 V_2 \delta t = p_2 v_2 \delta m$

The work done by the system is then $W = \dot{W}_s \delta t + p_2 v_2 \delta m - p_1 v_1 \delta m$

Substituting into the non-flow energy equation then gives:

$$\dot{Q}\delta t - \dot{W}_s \delta t - p_2 v_2 \delta m + p_1 v_1 \delta m = \delta m \Delta(u + \tfrac{1}{2}V^2 + gz).$$

Putting $h = u + pv$ and dividing by δt gives

$$\boxed{\dot{Q} - \dot{W}_s = \dot{m}\Delta\left(h + \tfrac{1}{2}V^2 + gz\right)}.$$

This is called the *Steady Flow Energy Equation* often abbreviated to SFEE.
$\dot{m}$ is the mass flow rate of fluid through the control volume.
Dividing by the mass flow rate gives:

$$\boxed{q - w_s = \Delta\left(h + \tfrac{1}{2}V^2 + gz\right)}$$

Take extreme care with units in these equations. It is easy to have *h* in kJ/kg and the other terms in J/kg.

Steady Flow Processes

It should be clear from the above that the heat transfer in a steady flow process is the same as in the equivalent non-flow process, but the work w_s differs from the non-flow work because of the 'flow work' terms. For calculations one usually can find the heat transfer as for a non-flow process of the same kind and then find the shaft work w_s using the steady flow energy equation.

Of particular interest is the steady-flow adiabatic process. One group of equipment includes compressors and pumps of all kinds that are driven by a shaft and are used to raise the pressure of a flowing substance. Another group includes reciprocating steam engines and turbines of various kinds, which have an output shaft driving some load and derive their work output by reducing the pressure of the working substance flowing through them. None of these devices has any deliberate heat transfer, and any heat transfer that does occur is trivial. Consequently the process taking place is idealised as adiabatic and steady flow. Furthermore on these devices the potential energy and kinetic energy terms in the steady-flow energy equation are often negligible compared with the shaft work. Eliminating those terms from the steady flow energy equation reduces it to:
$\dot{W}_s = \dot{m}\,\Delta h$

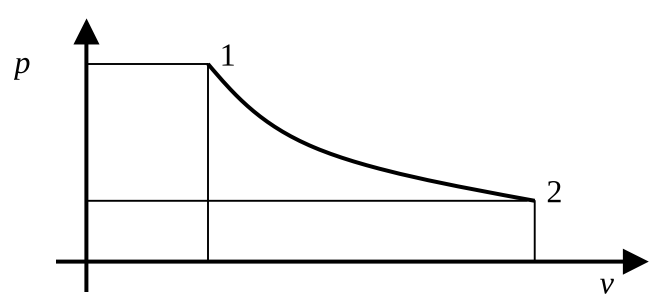
Similarly nozzles and diffusers have negligible heat transfer and no shaft work input or output. The change in kinetic energy is not negligible. For these devices $0 = \Delta\left(h + \frac{V^2}{2}\right)$.

Shaft Work

Start with the equation found above $W = \dot{W}_s\,\delta t + p_2 v_2\,\delta m - p_1 v_1\,\delta m$. Substituting $W = \dot{W}\,\delta t$ and dividing through by δt gives $\dot{W}_s = \dot{W} - p_2 v_2 \dot{m} + p_1 v_1 \dot{m}$. Then dividing by $\dot{m}$

$$w_s = w + p_1 v_1 - p_2 v_2 .$$

Remembering that $w = \int_1^2 p\,dv$ and sketching a *p*-*v* diagram:



w is the area under the process path. $p_1 v_1$ is the area of the rectangle between the origin and state 1. $p_2 v_2$ is the area of the rectangle between the origin and state 2, and w_s is the area to the left of the process path.

$$\boxed{w_s = -\int_1^2 v\,dp}$$

This is a most useful result to get, but if you find yourself evaluating the integral to solve a problem you are probably going the wrong way.